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PAVEMENT FRICTION TEST TIRE CORRELATION

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April 1975

Final Report

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16. Abstract

A correlation has been established between skid resistance measurements with the newly adopted test tire (ASTM E 501) and the previous standard test tire (ASTM E 249). The correlation is based on a large scale field test program, supported by laboratory tests on a high speed facility. Both tires respond in a similar way to changing test conditions, but tire E 501 is expected to measure about 4 percent higher than tire E 249 under standard test conditions at 40 mph. Prediction equations and the associated estimated variances are given for an all-inclusive correlation, as well as separate equations for three speeds (20, 40, and 60 mph) and separate equations for each of the four pavements used in this program. Limited tests on dry pavements also show generally higher readings with tire E 501.

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METRIC CONVERSION

This report uses U.S. Standard Units. For conversion to S.I. units the following factors should be used;

Length	1 inch	=	2.54 cm
Speed	1 mph	=	1.61 Km/hr
Temperature	F deg. F	=	(F-32)5/9 deg. C
Temperature Difference ...	1 deg. F	=	5/9 deg. C

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NOMENCLATURE AND ABBREVIATIONS

C		Tire Condition
D	=	$(x_1)249 - (x_1)501$
DF		Degree of Freedom
F_x		Tangential tire force
F_z		Normal tire force
G		Skid resistance - speed gradient
H		Water film depth
I		Time of day
P		Pavement type
R_e		Effective tire radius
SD		Standard deviation
SN		Skid number
SNX		Skid number measured with tire E 501
SNY		Skid number predicted for tire E 249
T	=	$(x_2)249 - (x_2)501$
V		Speed
a,b,c,		Coefficients in regression equations
n		Sample size (wheel rpm in laboratory tests)
s^2		Estimate of variance
x_1		Groove depth
x_2		Pavement temperature
x_3		Order of run
σ		Standard deviation
σ^2		Variance
σ_P^2		Variance contribution by pavement
σ_T^2		Variance contribution by tester

1. SUMMARY

The first pavement friction test tire (ASTM E 249) has been replaced by a new, somewhat larger tire (ASTM E 501). A correlation program was conducted in which skid resistance of four typical pavements was measured with each of the two tire types. The tests were made at several well defined conditions. Statistical analysis of the test results has led to the following conclusions:

i. The two tires do not differ appreciably in their performance as pavement test tires. The relative rates of wear have not been established.

ii. Tire E 501 gives readings about 4 percent higher than tire E 249 under standard test conditions (Fig. 1). This difference is of the same order of magnitude as the error in skid testing. Therefore, reversals must be expected, i.e., tire E 249 will occasionally give higher readings than tire E 501.

iii. Generally the effect of test variables, such as speed, water film thickness, temperature, inflation pressure, normal load and tire wear are the same for both tires. However, there is some evidence that tire E 501 is less sensitive than tire E 249 to variations in normal load, but has greater sensitivity to the effect of tire wear.

iv. Table 1 gives recommended conversion equations, where SNX and SNY are skid numbers for tires E 501 and E 249 respectively. For skid testing under standard conditions (ASTM E 274) equation c is recommended. The prediction variance under these conditions was found to be lowest (1.7 at a skid resistance level of 40 SN). This is based on a sample of eight skids and will be greater for a smaller sample.

Table 1. Recommended conversion equations.

	EQUATION	PAVEMENT TYPE	TEXTURE INCH	SPEED MPH
a	SNY = .977 SNX	ANY COMMON		10-70
b	SNY = .991 SNX	ANY COMMON		20
c	SNY = .957 SNX	ANY COMMON		40
d	SNY = .964 SNX	ANY COMMON		60
e	SNY = .986 SNX	PCC	.037	10-70
f	SNY = .924 SNX	JENNITE	.012	10-70
g	SNY = .997 SNX	CHIP SEAL/GRAVEL	.050	10-70
h	SNY = .918 SNX	JENNITE FLUSH SEAL/SAND	.023	10-70

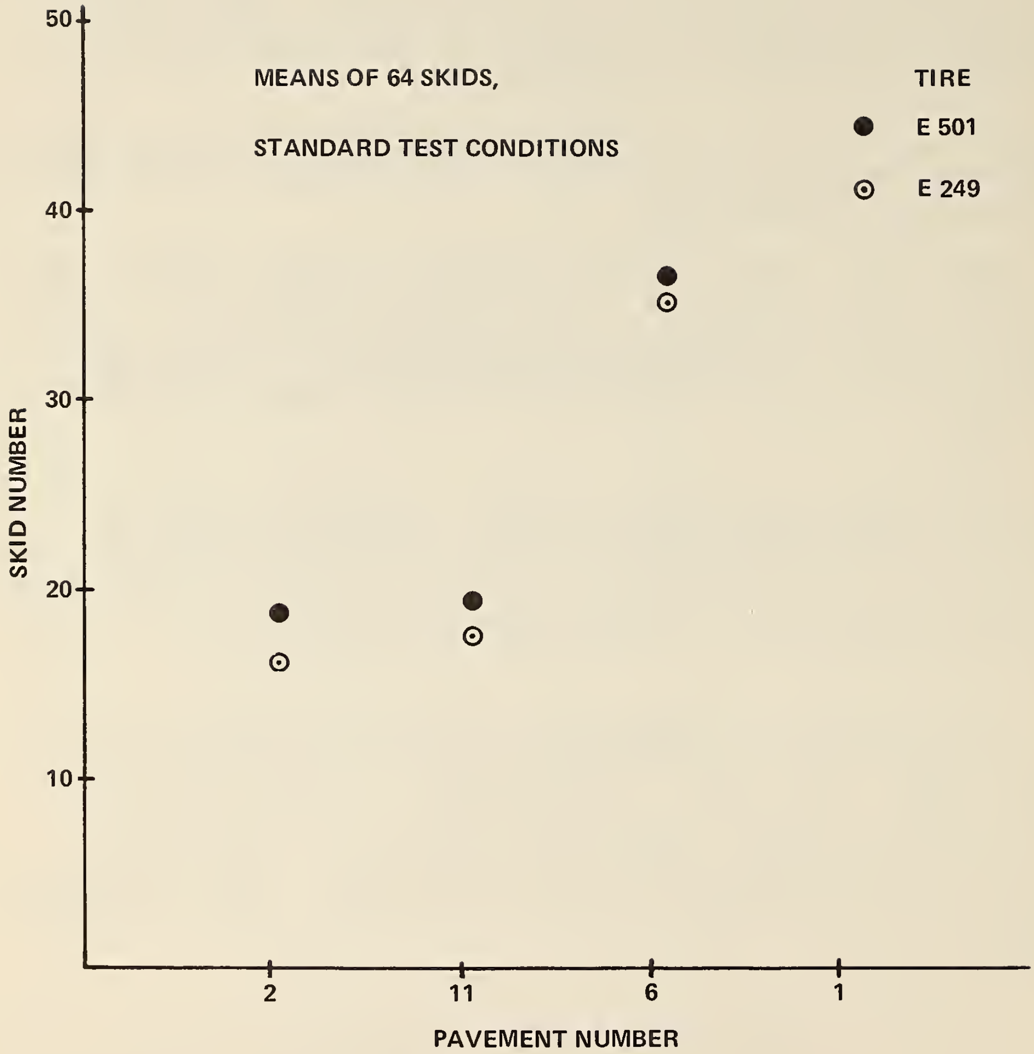


Figure 1. Mean skid resistance at standard test conditions.

v. Under dry conditions the difference between the two tires is somewhat greater, with tire E 501 reading 5 to 10 percent higher than tire E 249. Only limited data were available so that no formal correlation was made.

Based on these findings it is concluded that tire E 501 can replace tire E 249, using the same test conditions as before. Skid resistance requirements, in terms of SN_{40} should be increased by about 4 percent.

2. INTRODUCTION

For safe vehicle handling, friction between pavement and tires must be adequate. This requirement is easy to meet on dry pavements, but not on wet ones. For this reason pavements should be periodically surveyed (1)* to identify sections where pavement-tire friction under wet conditions is inadequate. Most highway departments have chosen the locked wheel skid tester for measuring pavement friction and this method has been standardized (2). At present about 40 states have locked-wheel skid testers and some have as many as five such units.

In recognition of the importance of skid resistance for traffic safety, an international conference was held in 1958 at the University of Virginia (3). As part of this conference a test program was conducted (4) in which eight pavement friction testers participated. Among the many conclusions, the need for a standard test tire was recognized (5) as an important factor in reducing the differences between test results.

The General Tire Company agreed to produce and supply this test tire, which was standardized by ASTM (6) under designation E 249. Production of this tire started in 1961 and will be discontinued with the adoption of a new test tire in 1974. During these 13 years more than 7000 tires were produced. Statistics supplied by the General Tire Company (7) cover the years 1967 to 1975. During this period 5414 tires were sold to: State Highway departments (44%), automotive industry (23%), tire test organizations, etc. (14.5%), universities (8%), tire industry (6%) and U.S. Government agencies (4.5%). The use increased steadily from 51 tires in 1967, to 725 tires in 1970 and 1346 tires in 1973. It then dropped off in 1974 (939 tires), presumably as the new tire became available.

In 1970 ASTM Committee E 17 decided to standardize a new 15-inch test tire, to replace the 14-inch E 249 tire, meet current compound and construction specifications and fit full size cars, using 15-inch wheels. This new test tire was produced by B. F. Goodrich Co. and was tested by several users in 1972. Test results showed a strong dependence on state of wear. The shoulder design and tread groove width were modified in an attempt to correct this defect. The modified tire became available in 1974 and was approved as ASTM standard test tire under designation E 501 (8). At the same time Committee E 17 recommended to withdraw the current standard for tire E 249 after 1 year.

*Numbers in parenthesis refer to references.

In order to assure continuity in skid testing and provide State highway departments and other users the means of comparing test data taken with the two tires, the FHWA Offices of Research and Development have undertaken a large scale correlation program, involving both field and laboratory tests. The field tests were conducted at the FHWA Field Test and Evaluation Center (9) at the Texas Transportation Institute. The laboratory tests were conducted at the CALSPAN tire test facility TIRF (10). Details of the test program are given in Appendix A. Data processing and analysis was done by the authors of this report. The objective was to establish a correlation between skid resistance measurements taken in the past with the standard test tire E 249 and future measurements which will be made with the new test tire E 501. The correlation was to be established over a range of conditions as may be encountered in skid testing. Also, the reliability of the correlation (variance of the predictions) was to be determined.

The analysis was limited to meet the primary objectives of this program, but the accumulated data can be used for investigating other aspects of interest in skid testing, some of which are presented in Chapter 5.

3. EXPERIMENTAL PROGRAM

The primary objective of the test program was to establish a correlation between the two test tires. Many years of experience in skid resistance measurements has shown that the variability in skid testing is relatively large and depends on many factors (11). It was therefore necessary to include at least some of the principal factors as variables in the test program. Five such factors were selected and are listed and briefly discussed below.

a. Pavements

Four pavements were selected to span a range of skid resistance and texture. These are described in Appendix B. The limited length of the test surfaces permits only one wheel-lock up per pass. Thus, to perform the programmed eight repeat runs, the tester had to make four passes in each direction.

b. Speed

Tests were conducted at 20, 40, and 60 mph. Forty miles per hour is the standard test speed (2). Tests at 20 and 60 mph were expected to respectively attenuate and amplify the effect of wetness and tire wear on skid resistance.

c. Water Depth

In addition to the standard nominal water depth of 0.020 inches, a depth of 0.033 inches, the maximum obtainable with the available equipment, was also used. In the laboratory, tests were run on a range of water depths between zero (dry) and 0.060 inches.

d. Tire Condition

The tires were used in two extreme conditions: (i) new and (ii) shaved to below the wear line. Groove depth was measured. In the laboratory tests an intermediate groove depth was also used.

e. Time of Day

To cover as wide a temperature range as possible, tests were conducted in the morning and repeated in the early afternoon. Pavement, tire and ambient temperatures were recorded.

Laboratory tests will be discussed in a separate section, while the following applies only to the field tests.

The five factors, having 4 levels of pavement, 3 levels of speed and 2 levels each of water depth, tire condition and time of day, give 96 possible test conditions. Each test condition was replicated four times, using different tires, for a total of 384 tests per tire type. Each such test consisted of eight consecutive runs, for a total of 3072 planned skids per tire type.

Actually a total of 3840 skid resistance measurements were made with each tire type. The data were processed in five sets according to the test conditions listed in Table 2.

Table 2. Test conditions for the five data sets, each with 768 skids per tire type.

SET NO.	WATER DEPTH	TIRE COND.	PAVEMENT	SPEED	TIME OF DAY
	H (inches)	C	P	V (mph)	I
1	H1=0.020	C1=new	all	all	all
2	H1=0.016	C1=new	all	all	all
3	H1=0.020	C2-worn	all	all	all
4	H2=0.033	C2=worn	all	all	all
5	H2=0.033	C1=new	all	all	all

The conditions in Sets 1 and 2 were almost identical, because of improper setting of the water pump. In the initial analysis each data set was treated separately and all data were used. In the final, combined analysis, set No. 2 was omitted to maintain a balanced data base, in accordance with the original plan. Thus the correlation equations are based on 3072 skid resistance measurements per tire type.

The test plan was a compromise between complete randomization and a systematic sequence, for efficient use of the test facility and an 8-hour work day. To further reduce systematic errors, the same tester, procedure and crew were used throughout the test program.

The original plan called for three procedural steps of data analysis (Appendices C and D):

a. Computation of means and standard deviations of each group of 8 runs, as well as an analysis of order-of-run effects within each group.

b. Analysis of variance to determine the significance of different test variables and effect of covariates (groove depth, temperature and order-of-run) on each test variable.

c. Correlation between the two test tires.

These three steps were considered necessary for eliminating variables of lesser significance and reducing the complexity of the calibration model.

The test program provided a large data base which can yield valuable information on skid testing in general. Therefore, in addition to the primary objective of establishing a correlation between the two test tires, regressions were developed on variables of interest, such as speed and temperature.

In planning the tests, a choice had to be made between spacing the individual runs across the pavement or repeating all runs on the same wheel track. The former option resembles conditions in inventory testing and includes the effect of pavement variability. The latter option was adopted, however, even though the test results may be biased because of the prewetted pavement condition. This was not expected to affect the correlation, since both tires would be tested under the same conditions. On the other hand, this option might reduce the effect of pavement variability. To reduce variability between the first "dry" and consecutive "wet" skids, each test was preceded by two prewetting runs, with the test tire free rolling.

4. TEST RESULTS AND ANALYSIS

4.1 Preliminary Analysis

In the preliminary analysis the mean skid resistance of eight runs and the standard deviations within each group of eight runs were computed. To determine if prewetting introduces systematic errors, the data were analyzed to establish if the order-of-run had a significant linear cumulative effect.

A formal consolidated F-test on all data groups showed that the order-of-run effect was insignificant and therefore all further analyses were made on mean skid numbers (averages of eight). This test was conducted at the 0.05 level of significance for each tire type, as shown in Appendix C. As an alternative verification, an individual F statistic was calculated for the linear order-of-run effects in each group. The resulting set of F values indicated significance only 52 out of 960 times (5.4%), with 14 (1.5%) showing a systematic increase in skid numbers and 38 (3.9%) showing a systematic decrease.

Means and variances were tabulated for each data set, which consisted of four replicates under identical conditions. Figures 2 and 3 show typical outputs. (The complete data are given in Appendix E.) Each cell entry in Fig. 2 represents the mean of eight measurements, while the corresponding entry in Fig. 3 is the usual estimate of variance, given by

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} \qquad (1)$$

Grouped averages by speed, site (pavement) and tire type are also given. Five such data sets were available for a total of 480 data pairs. These are summarized by tire type and pavement in Table 3, which shows that the mean skid numbers and variances are similar for both tire types. Thus the paired data are amenable to unweighted regression analysis.

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 4
MEAN

SITE	REPS SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	24.300	23.562	21.800	22.587	23.062	24.300	24.800	23.175	23.950	24.056
		25.925	22.587	22.950	22.275	23.434	25.437	20.850	25.200	24.450	23.984
2	40	12.687	15.700	14.275	13.950	14.153	15.800	17.100	15.525	13.950	15.594
		14.250	15.425	14.600	14.362	14.659	15.787	15.900	16.150	15.300	15.784
	60	10.987	11.262	9.862	10.800	10.728	11.500	10.925	11.225	12.012	11.416
		11.500	11.950	10.637	10.800	11.222	11.712	12.575	11.725	11.262	11.819
	20	23.350	19.412	21.175	21.662	21.400	26.312	20.887	21.800	22.125	22.781
		21.487	20.737	22.112	20.425	21.191	22.600	22.812	26.800	21.525	23.434
11	40	15.437	15.200	15.525	16.050	15.553	17.000	16.400	17.237	16.500	16.784
		15.787	15.787	16.262	15.500	15.834	16.275	16.400	18.937	17.550	17.291
	60	12.562	11.750	11.962	11.400	11.919	12.212	11.862	12.800	13.200	12.519
		11.837	12.550	11.350	10.875	11.653	12.812	12.575	13.537	11.100	12.506
	20	51.012	49.437	48.650	50.987	50.022	48.012	49.437	48.387	48.062	48.475
		51.612	49.575	49.437	47.900	49.631	52.562	51.937	49.437	48.387	50.581
1	40	38.662	38.500	38.562	37.087	38.203	40.075	39.712	37.512	39.800	39.275
		39.637	38.725	37.575	37.087	38.256	39.025	40.500	37.562	38.300	38.847
	60	32.050	30.250	29.425	27.950	29.919	35.925	31.487	27.550	30.875	31.459
		33.525	32.550	30.675	30.300	31.762	32.687	35.337	28.800	31.887	32.178
	20	37.437	43.562	32.675	35.987	37.416	33.437	38.375	33.425	35.512	35.187
		33.112	36.962	33.550	33.550	34.294	33.500	36.825	34.425	37.525	35.569
6	40	28.675	32.462	30.050	30.000	30.297	29.775	31.800	30.987	30.612	30.794
		29.512	32.175	28.925	28.600	29.803	30.425	32.300	29.175	29.862	30.441
	60	30.175	27.062	27.925	26.412	27.894	29.550	28.150	27.300	27.000	28.000
		25.425	27.800	27.925	28.300	27.362	27.612	30.300	27.800	26.462	28.044

BY SPEED	20	40	60
	32.782	25.098	20.650

BY SITE	2	11	1	6
E249	16.210	16.258	39.632	31.178
E501	17.109	17.553	40.136	31.339

BY TIRE	
E249	25.819
E501	26.534

Figure 2. Typical summary printout by mean skid number.

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 4
STANDARD VARIANCES

		TIRE E249					TIRE E501				
REPS		1	2	3	4	MEAN	1	2	3	4	MEAN
SITE	SPEED										
	20	2.285	6.965	0.286	2.539	3.019	4.000	0.571	2.125	0.592	1.822
		14.410	2.639	4.006	2.488	5.886	9.823	2.146	21.734	12.557	11.565
2	40	0.816	1.263	2.411	2.649	1.784	1.734	4.883	0.982	1.826	2.356
		1.666	0.485	2.294	3.551	1.999	3.044	1.449	1.177	1.234	1.726
	60	0.438	1.186	0.880	1.543	1.012	2.020	1.605	3.488	4.724	2.959
		4.340	1.246	2.280	1.131	2.249	2.078	0.839	1.345	2.817	1.770
	20	10.003	0.816	2.125	4.177	4.280	23.844	8.087	1.714	5.060	9.676
		2.636	1.717	7.124	2.488	3.506	5.988	2.801	13.142	2.351	6.071
11	40	1.248	1.789	1.839	4.912	2.447	1.603	2.080	2.831	1.235	1.937
		0.981	0.438	0.751	0.686	0.714	5.522	1.334	6.694	1.415	3.741
	60	1.623	1.191	0.503	0.720	1.009	2.864	0.380	1.334	2.777	1.839
		1.080	1.034	0.920	0.319	0.838	3.664	3.105	3.706	1.646	3.030
	20	3.033	9.065	2.807	5.766	5.168	4.599	3.482	1.830	7.456	4.342
		13.262	3.450	3.658	14.488	8.714	6.545	12.153	3.782	24.210	11.672
1	40	5.578	1.143	3.454	2.286	3.115	2.089	8.658	16.617	7.406	8.693
		9.539	10.094	7.547	5.446	8.156	13.146	14.570	8.912	5.244	10.468
	60	7.641	3.720	2.553	5.281	4.799	3.267	5.287	14.499	14.107	9.290
		6.381	7.927	16.695	2.095	8.275	12.645	6.193	15.999	5.787	10.156
	20	3.225	16.104	6.981	3.532	7.461	2.381	8.554	5.410	5.927	5.568
		7.658	25.264	8.784	10.071	12.945	6.333	18.318	9.123	11.478	11.313
6	40	4.981	3.309	2.214	3.877	3.595	5.561	1.142	4.044	3.556	3.576
		3.267	1.410	9.552	2.330	4.140	5.695	4.285	4.553	1.769	4.076
	60	1.982	2.782	0.696	3.661	2.280	1.070	4.263	0.285	3.220	2.210
		2.267	3.429	2.696	2.600	2.748	2.890	4.857	7.428	2.414	4.397

BY SPEED	20	40	60
	7.063	3.908	3.679

BY SITE	2	11	1	6
E249	2.658	2.133	6.371	5.528
E501	3.700	4.382	9.103	5.190

BY TIRE	
E249	4.172
E501	5.594

Figure 3. Typical summary printout by variance.

Table 3. Summary of mean skid numbers (SN), variances (σ^2), standard deviations (σ) and percent standard deviations ($100\,\sigma$/SN).

		PAVEMENT							
		2		11		1		6	
SET	TIRE	249	501	249	501	249	501	249	501
1	SN	17.93	20.48	17.93	19.40	42.09	44.72	37.38	37.61
	σ^2	2.64	3.40	1.91	2.18	2.60	2.46	7.55	8.35
	σ	1.63	1.84	1.38	1.48	1.61	1.57	2.75	2.89
	100σ/SN	9.1	9.0	7.7	7.6	3.8	3.5	7.4	7.7
2	SN	15.98	20.16	16.39	18.87	42.97	45.14	35.42	36.68
	σ^2	3.98	2.98	2.57	2.29	3.78	3.56	7.87	5.84
	σ	2.00	1.73	1.60	1.51	1.95	1.92	2.80	2.92
	100σ/SN	12.5	8.6	9.8	8.0	4.5	4.3	7.9	6.6
3	SN	17.54	19.04	16.38	17.71	41.81	42.68	31.90	32.93
	σ^2	3.67	3.53	2.55	3.30	4.55	5.98	4.64	6.61
	σ	1.92	1.38	1.60	1.82	2.14	2.45	2.16	2.57
	100σ/SN	10.9	9.9	9.8	10.3	5.1	5.7	6.8	7.9
4	SN	16.21	17.11	16.26	17.55	39.63	40.14	31.18	31.39
	σ^2	2.66	3.70	2.13	4.38	6.37	9.10	5.53	5.19
	σ	1.63	1.93	1.46	2.10	2.52	3.02	2.36	2.28
	100σ/SN	10.1	11.3	9.0	12.0	6.4	7.5	7.6	7.3
5	SN	17.11	19.01	17.89	19.40	42.48	49.30	32.53	33.78
	σ^2	2.58	3.75	3.53	2.31	8.18	6.91	9.87	10.26
	σ	1.61	1.94	1.88	1.52	2.86	2.63	3.14	3.20
	100σ/SN	9.4	10.2	10.5	7.8	6.7	5.9	9.7	9.5
ALL	SN	16.95	19.16	16.97	18.59	41.80	43.40	33.68	34.37
	σ^2	3.11	3.47	2.54	2.89	5.10	5.60	7.09	7.25
	σ	1.76	1.86	1.60	1.70	2.26	2.37	2.66	2.70
	100σ/SN	10.4	9.7	9.4	9.2	5.4	5.5	7.9	7.9

By a preliminary examination of mean skid numbers and variances over all factor levels and replications, some initial comparisons can be made. In Table 4 skid numbers and variance for the two tire types are compared. It can be seen that when single data pairs are compared, the E 501 tire gives higher readings about 80 percent of the time. This increases to more than 90 percent when based on four replicates and to 100 percent of means over all measurements on each of the four pavements. Thus, reversals do occur, but for a large enough sample, the skid resistance measured with the new tire is generally above that measured with the E 249 tire.

Table 4. Comparixon of skid numbers and variances for tires E 501 (X) and E 249 (Y).

DATA	SKID NUMBER, SN			VARIANCE σ^2		
	SNX>SNY	SNX = SNY	SNX<SNY	$\sigma^2X>\sigma^2Y$	$\sigma^2X=\sigma^2Y$	$\sigma^2X<\sigma^2Y$
480 PAIRS OF MEAN SN[(1)]	390 (80%)	7	83	276	2	202
120 PAIRS OF MEANS [(2)]	109 (90%)	0	11	71	0	49
5 SETS ON 4 PAVEMENTS	20 (100%)	0	0	11	0	9

(1) BASED ON 8 SINGLE RUNS

(2) BASED ON FOUR REPLICATES

The comparison in Table 4 also shows that the variances with tire E 501 are greater in 58 percent of the total number of data pairs. The differences are, however, small. The pooled standard deviations (Table 9) are 2.19 and 2.11, the overall mean skid numbers are 28.88 and 27.35 (Appendix E) for tires E 501 and E 249 respectively. Dividing the standard deviation by the SN results in a "percent standard deviation," equal to about 7.6 percent of the mean skid number for both tires. Thus, the variance of the mean skid testing is about the same with both tires. It would be of interest to determine how great a contribution the tires make to the total variance.

Based on information in Reference 11 the total variance in skid testing is between 1.5 and 2.5 times the tester variance. Since a single skid tester was used in this program, we are justified to use the smallest tester variance, that is total variance divided by 2.5. This gives a tester standard deviation of 1.36 SN (skid numbers) including the tire contribution. The pavement contribution varies between 1.04 and 2.30 on an absolute basis and between 4.4 and 6.8 on a percentage basis (Table 5). As could be expected, percent standard deviations have a narrower range than absolute ones. Pavement No. 6 has the largest standard deviation on both scales. It appears thus that the measurement error is the same with both tires, but will change with pavement type and skid resistance level.

Also of interest are the effects of tire wear and water depth. These have been graphically presented in Figure 4 which show SN versus speed. The plotted curves are based on the first data set (new tires and standard 0.020 inch water depth). The cross hatched regions show the confidence limits, which were computed using 0.8 times the standard deviation for each speed and pavement (from Fig. 12 in Ref. 11 for a sample size of 8). Increased water depth by itself is seen to have the smallest effect, in most cases smaller than tire wear. The largest drop in SN occurs when the worn tire is used with increased water depth.

Figure 5 shows matrices of the percentage drop in SN among four different test conditions, water depth, worn tire and a combination of these. Comparing the overall drop for the two tires (lower right hand field), we see that the effect of thicker water films was about the same (4.61 and 4.05 percent on E 249, 4.71 and 5.15 percent on E 501). The effect of tread wear on skid resistance measurement, which was the reason for modifying the initial design (as discussed in the Introduction) is still different for the two tires (6.66 and 6.11 percent on E 249, 8.48 and 8.89 percent on E 501). This drop of close to 9 percent is less than the 14 percent drop found previously in limited tests on four pavements (12). Thus some improvement has been achieved, but the effect is still greater with the new standard tire than with tire E 249. Further improvement may be expected when the groove width is increased to meet the original specifications (Section 6.2).

Table 5. Pooled variances and (standard deviations) on the four pavements for both tire types (mean of 8 runs).

SET	PAVEMENT			
	2	11	1	6
1	3.02 (1.73)	2.05 (1.43)	2.53 (1.59)	7.95 (2.82)
2	3.48 (1.87)	2.43 (1.56)	3.67 (1.93)	6.86 (2.62)
3	3.60 (1.90)	2.93 (1.73)	5.77 (2.30)	5.63 (2.38)
4	3.18 (1.79)	3.26 (1.81)	7.74 (2.78)	5.36 (2.32)
5	3.17 (1.78)	2.92 (1.71)	7.55 (2.75)	10.07 (3.18)
MEAN, σ^2	3.29 (1.83)	2.72 (1.65)	5.35 (2.32)	7.17 (2.68)
OVERALL	4.63 (2.15)			
TESTER = OVERALL/2.5, $\sigma_T^2 =$ 1.85 (1.36)				
PAVEMENT $\sigma_P^2 = \sigma^2 - \sigma_T^2$	1.44 (1.20)	1.07 (1.04)	3.50 (1.87)	5.32 (2.30)
SN	18.60	17.78	42.60	34.03
100 σ_P/SN	6.5	5.8	4.4	6.8

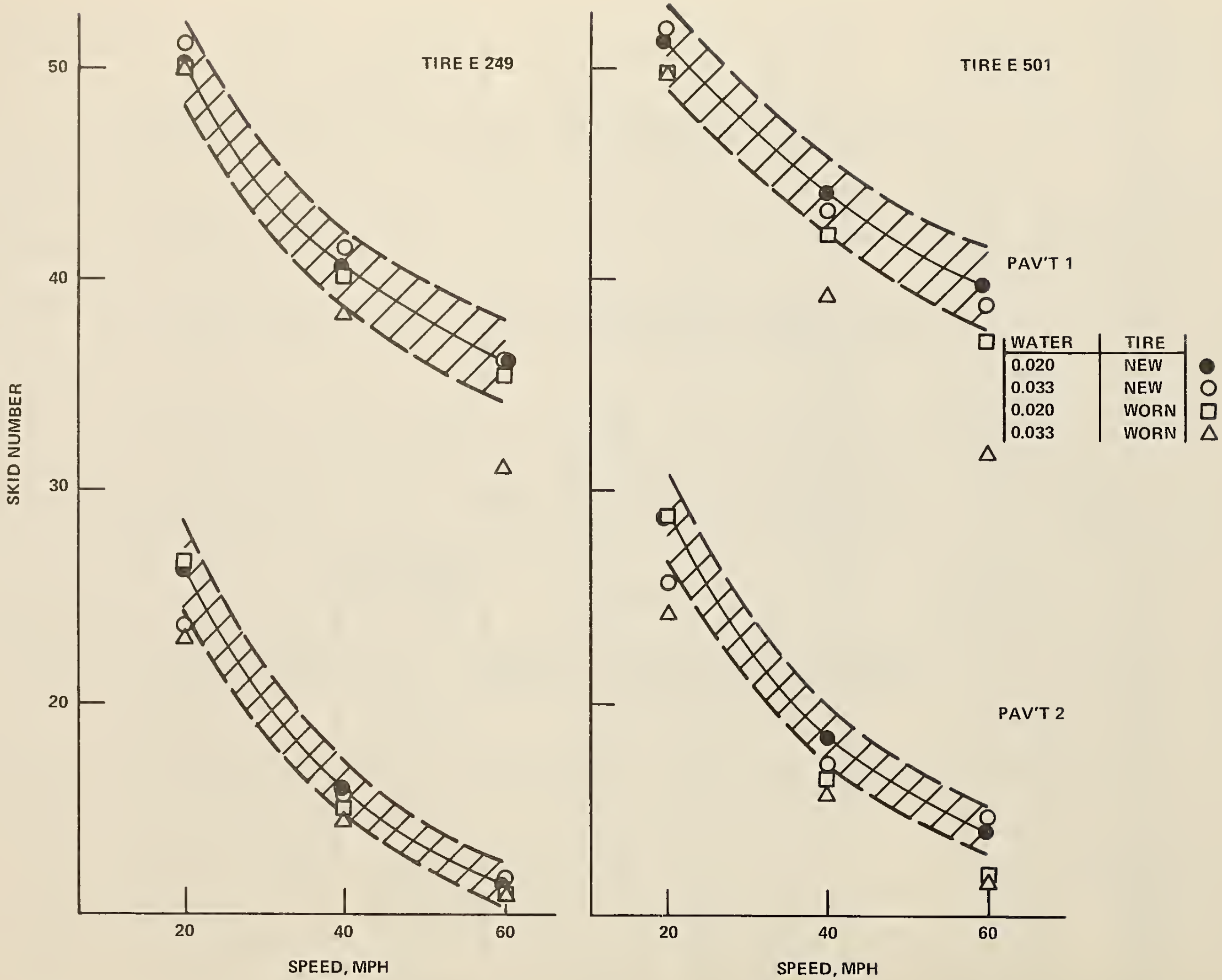


Figure 4. Skid resistance versus speed, and changes with increased water depth and reduced groove depth.

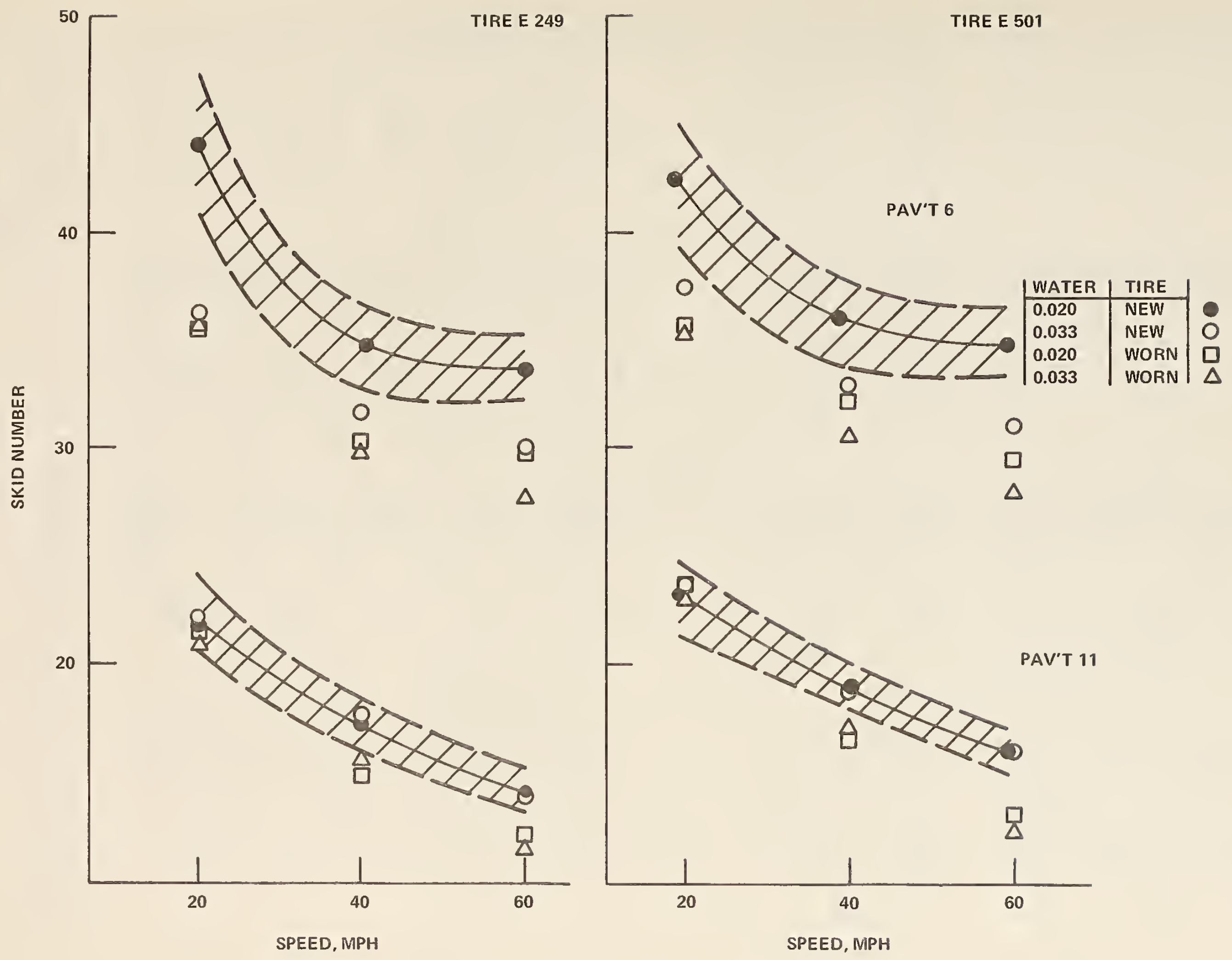
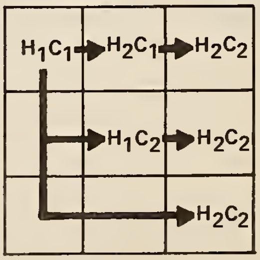


Figure 4. (continued).

PAVEMENT NUMBER	SPEED, MPH											
	20			40			60			ALL		
2	0	10.24	1.40	0	2.00	8.22	0	-4.79	8.81	0	4.57	5.26
		1.26	12.6		5.93	4.38		4.53	-0.09		2.12	7.69
			11.50			10.05			4.77			9.59
11	0	2.31	3.05	0	-2.38	11.00	0	1.47	16.26	0	0.22	9.11
		2.24	2.25		12.49	-4.11		14.14	3.91		8.64	0.73
			4.44			8.89			17.49			9.31
1	0	-1.86	2.04	0	-2.35	7.48	0	2.11	12.49	0	-0.93	6.71
		-0.22	0.44		0.79	4.54		1.83	12.73		0.67	5.21
			0.22			5.30			14.33			5.84
6	0	17.17	1.02	0	1.24	4.42	0	11.40	7.65	0	12.97	4.15
		18.63	-0.48		12.30	1.09		11.96	7.06		14.66	2.26
			18.02			13.25			18.18			16.59
ALL	0	6.75	1.81	0	2.00	7.24	0	4.44	11.00	0	4.61	6.11
		5.76	2.83		7.10	2.15		7.54	8.02		6.66	4.05
			8.44			9.09			14.95			10.44

PAVEMENT NUMBER	SPEED, MPH											
	20			40			60			ALL		
2	0	10.91	5.99	0	7.97	8.19	0	-0.84	19.25	0	7.18	9.99
		0.35	16.22		10.61	5.42		16.60	2.02		7.03	10.14
			16.25			15.46			17.41			16.46
11	0	-2.15	2.53	0	0.99	9.98	0	2.08	19.60	0	0.00	9.54
		-1.51	1.91		13.60	-3.15		17.81	4.21		8.71	0.90
			0.43			10.88			21.27			9.54
1	0	-1.08	3.69	0	1.39	9.44	0	3.11	16.98	0	0.94	9.39
		2.83	-0.18		4.39	6.60		7.00	13.51		4.56	5.95
			2.65			10.70			19.57			10.24
6	0	11.57	5.33	0	7.42	7.41	0	11.36	9.35	0	10.21	7.23
		16.21	0.08		9.83	4.94		14.91	5.56		13.78	3.36
			16.28			14.28			19.64			16.69
ALL	0	4.80	4.37	0	4.23	8.73	0	5.13	15.36	0	4.71	8.89
		5.49	3.68		8.54	4.44		12.59	8.14		8.48	5.15
			8.96			12.60			19.70			13.19

Figure 5. Percentage drop in SN, (negative values represent increases).
Upper table-tire E 249, lower table-tire E 501.



4.2 Analysis of Variance

The purpose of the analysis of variance (ANOVA) is to determine the effects of each of the five factors (listed in Chapter 3) on the mean skid numbers. Since two of those five, groove depth and temperature, were measured, the qualitative descriptors "Tire Condition" and "Time of Day" could be replaced by quantitative measures, the covariates. This was done in an analysis of co-variance (CO-ANOVA) for three factors, pavement (4 levels), speed (3 levels) and water depth (2 levels), and two covariates, groove depth and wet pavement temperature. The analysis is performed for each tire type as well as for both tires combined, in which case tire type becomes an additional factor (2 levels).

The basis for use of the various statistical techniques employed here is given in Appendix C. The preliminary analysis (Section 4.1) showed that order-of-run effects on the observed SN were insignificant. Thus the mean of eight runs (observations) may be assumed to have the same expected value as any of the eight individual runs, since all experimental conditions were identical. This result effectively eliminated order-of-run as another covariate and also allowed us to compress the data by a factor of eight.

The analysis provides a measure of the contribution of each variable to the measured skid numbers, as well as a measure of the experimental error. These error variances of the two tire types are compared, and are also compared to the means of the "within" variances (the variances within each group of eight runs), obtained in the preliminary analysis. Thus, answers to the following questions are sought:

a. Which of the two standard test tires is more variable, the E 249 or the E 501?

b. What are the significant interactions between variable factor effects?

c. Are the error variances the same at different speeds, on different pavements?

d. Does the increase of water depth from the standard of 0.020 inches to 0.033 inches have a significant effect on the measured SN? If such an effect exists, is it the same for both tires?

e. Do groove depth and temperature have significant effects on the measured SN? Does the measured SN increase or decrease with increasing groove depth or increasing temperature?

f. How do the error variances compare in the "between" mean analysis (ANCVA) and the "within" mean analysis of Section 4.1?

To answer these questions, the data for each tire were first processed on the Analysis of Covariance BMDO3V computer program (Appendix D). This program does not provide estimates of the main and interaction effects, so that, for instance, the second question in (d) above could not be immediately answered. Therefore a second program, OMNITAB (Appendix D), was also used. This, special, user-oriented computing system, has a routine to automatically provide the desired outputs. But most questions could be answered by the BMDO3V analysis, although to answer question (c), separate analyses by speed or pavement are needed.

Identification of significant interactions (Question b) is important. The following definition will clarify the term: "Interaction measures the failure of the effect of variable A to be the same at different levels of variable B." For example, is the observed difference in the SN value at the two water depths the same on different pavements or at different speeds, or are there significant differences?

Answers to most of these questions are provided below. These will be seen to affect the modelling for the calibration procedure, discussed in Section 4.4.

4.3. Consolidated Analysis

The analysis is based on data sets 1, 3, 4 and 5 of Table 2 (as explained in Section 4.1). The 3072 observations per tire were compressed into 384 means, based on the findings of the preliminary analysis. Table 6 is a summary of mean square values of sources of variations and interactions. For each source an F-test was made at the 5 percent level and the appropriate degree of freedom.

Table 6. Summary of analysis of variance (between means).

SOURCE		D.F.	MEAN SQUARES			
			E 249	E 501	AVERAGE (2 TIRES)	BOTH TIRES
WATER DEPTH	H	1	119.79	146.71	133.25	133.00
PAVEMENT	P	3	12,682.78	11,796.92	12,239.85	12,237.30
SPEED	V	2	4,349.23	3,859.99	4,104.61	4,101.08
	HxP	3	30.18	22.51	26.34	25.10
	HxV	2	10.83	1.33	6.08	4.73
	PxV	6	128.34	104.28	116.31	113.01
	HxPxV	6	13.11	18.38	16.05	15.72
ERROR VARIANCE		358	3.38	2.97	3.17	(718D.F.) 3.18

D. F. degree of freedom

All main effects and some interactions were highly significant. As could be expected, pavement type and speed had predominant effects on skid resistance, while the pavement-speed interaction effect is also rather large. Level of water depth and its interaction with pavement type were significant, but not the other interactions.

Main effects and interactions are seen to be of the same order of magnitude for both tires. The last column in Table 6 is taken from the combined analysis for both tires. Care must be taken in interpreting these results, since, with tire type as an additional treatment factor, the model is changed and the block sizes of all effects are now twice as large as in the individual analyses. Consequently the expectations of their associated mean squares are twice as large. To compare the mean squares, the last column in Table 6 lists the computed results divided by a factor of two. The results show good agreement with the column for the average, which leads to the conclusion that the main effects and their significant interactions are about the same for both tires.

The strong effect of pavement-speed interaction suggests the need for separate analyses. These were performed as part of the calibration and will be discussed in Section 4.4.

The error variance of tire E 249 is seen to be greater than that of tire E 501 (last line in Table 6). The significance of this difference can be determined by a two-sided F-test. The critical value for the given degree of freedom is obtained by harmonic interpolation and is 1.11, for the upper 2.5 percent. The ratio 3.38/2.97=1.138 and thus exceeds the critical value. Thus the difference between the error variances is significant, so that skid resistance measurements with tire E 249 may be expected to have a slightly greater experimental error than with tire E 501. It should be emphasized here, that this difference in variance applies to mean SN's. The "within" variance has been shown to be about the same for both tires (Section 4.1).

The effect of covariates, groove depth and temperature, can be examined from Table 7. Slopes and t-values are computed

Table 7. Linerar regression coefficients for covariates groove depth and temperature.

TIRE	GROOVE DEPTH			WET PAVEMENT TEMPERATURE		
	SLOPE b	S.D.	t*	SLOPE b	S.D.	t*
E 249	8.700	0.909	9.568	-0.025	0.0106	-2.342
E 501	10.402	0.720	14.457	-0.023	0.0099	-2.303
BOTH	9.694	0.568	17.056	-0.024	0.0073	-3.286

by the CO-ANOVA routine. The listed standard deviations are obtained from the relation b/(S.D.) = t. All t-values are seen to be significant. To test if there is a significant difference between the tires with regard to the effects of groove depth and temperature, the null hypothesis of equal effects is tested. For the large degree of freedom, the observed slopes may be considered normally distributed with known variance. An approximate test is given by

$$u = (b_1 - b_2)/[(S.D.)^2_{b_1} + (S.D.)^2_{b_2}]^{½} \qquad (2)$$

Substituting the appropriate values gives
u = 1.16 for groove depth and 0.14 for temperature. The absolute value of u at the 0.05 level is 1.96 and therefore the hypothesis is accepted, namely there is no siginificant difference between the tires regarding the effect of groove depth and temperature. This finding has to be judged, however, against the large experimental error, reflected in the tabulated standard deviations of the slopes (Table 7), which are, in the case of temperature of the same order of magnitude as the slopes. The slope for groove depth is positive, therefore a worn tire will measure lower skid resistance under wet conditions. The slope for temperature is negative, confirming previous findings that the skid resistance generally decreases with increasing temperature.

Significance of the effect of water depth was established in the analysis of covariance (Table 6). The BMD03V program does not, however, provide information on the magnitude or direction of this effect. To determine these, the regression fit capability of OMNITAB was utilized. The triple interaction HPV, with its six degrees of freedom, was omitted from the analysis, but all main effects, first order interactions and covariate effects were included. The following values were obtained for the effect of water depth (Table 8).

Table 8. Regression coefficients for level of water depth $H = H_1 = -H_2$.

TIRE	H	S.D. of H	D.F.
E 249	0.735	0.157	364
E 501	0.831	0.149	364

S.D. - standard deviation
D.F. - degree of freedom

With the large number of degrees of freedom, a normal distribution can be assumed and the same test statistic (Eq.2) applied. From the results it can be inferred that the effect of increased water depth is the same for both tires. Also, since the positive value of H is associated with the lower water depth and the negative value with the greater water depth, we conclude that skid resistance decreases with increasing water depth. The mean difference, for both tires and all pavements and speeds is approximately 1.6 SN, ($H_1 - H_2 = 2H$). The contribution of the HxP interactions, although determined to be significant (Table 6), are relatively small. From the OMNITAB regression equations, these contributions have been found to be in all cases less than 1 SN, while the contributions of the HV interactions were less than 0.5 SN.

We also compare the error variances obtained in the "between" mean analysis with "within" mean variances obtained in the preliminary analysis (Section 4.1). Statistical theory shows that, if no other components of variance are introduced, the variance of means of n observations should be 1/n times the variance of an individual observation.

The model in the analysis of variance assumed that the variances for each cell of eight observations are homogeneous. If this model is correct, then the estimated variance between means should be smaller by a factor of eight than the variance within means computed in the preliminary analysis.

Table 9 summarizes the "within" variances and lists the "between" variances as obtained in the analysis of variance. Several interesting observations can now be made. First, there is a reversal in the relative magnitude of the variances. The new tire, E 501, exhibits a larger "within" variance, but a smaller "between" variance than tire E 249. The variances, however, are of the same order of magnitude for both tires and the small differences are probably attributable to uncontrollable experimental variations. Secondly, the "between" variances, although somewhat smaller than the "within" variances, are not smaller by a factor of eight. This can only be explained by error sources that occurred, but were not accounted for in the analysis. The largest error source in these tests, is probably the transverse and longitudinal variability of the pavements. Also some seasonal variations in the surfaces due to environmental effects may have occurred, since the tests extended over a period of 3 1/2 months (Sept. to Dec. 1974).

Table 9. Comparison of "within" and "between" error variances.

SPEEDS / PAV'T	TIRE E 249			TIRE 501		
	20	40		20	40	60
2	5.80	1.99	1.54	6.35	2.52	1.54
11	4.21	2.00	1.40	5.09	1.92	1.67
1	5.19	4.35	5.80	5.23	5.07	6.45
6	12.60	5.15	3.54	11.71	5.95	4.12
MEANS	6.94	3.37	3.07	7.09	3.87	3.45
POOLED WITHIN VARIANCE	4.46			4.80		
STAND. DEV.	2.11			2.19		
POOLED BETWEEN MEAN VARIANCE,	3.38			2.97		
STAND. DEV. *	1.84			1.72		

* FROM TABLE 6

Another observation worth noting is that the error variances are, except on pavement No. 1 (portland cement concrete), highest at 20 mph and much lower at the two higher speeds. This has been attributed to the usually greater skid resistance-speed gradients at low speeds (Ref. 11, p. 25). Therefore for the same deviation from the desired test speed, the spread of measured skid resistance will be greatest at the lowest speed.

Finally, the effect of variability among tires of the same type was investigated. All tests were replicated four times, using different tires. An analysis of variance, for each tire type, in which replication was treated as a factor, showed no significant differences among the four replications as far as skid resistance measurements are concerned. It should, however, be pointed out that the test tires were from a single production batch, so that no conclusions can be drawn regarding the variability between production batches.

4.4 Calibration of Skid Resistance Data

In this the third part of the analysis, various sets of equations are derived relating SN values of tires E 249 and E 501. Thus, when skid resistance is measured with the new test tire, the equivalent skid resistance for tire E 249 can be computed by using the appropriate equation. This type of correlation procedure is referred to as statistical calibration. It tells how to set the scale of one quantity so as to "read off" (via a regression equation) the desired value of the dependent quantity).

The full calibration model is given by Eq. 3.

$$SNY = a_0 + a_1\ SNX + a_2 D + a_3 T \qquad (3)$$

where

SNY = the predicted skid resistance for tire E 249,
SNX = the measured skid resistance with tire E 501,
$D=(X_1)_{249} - (X_1)_{501}$ (inches), $T=T_{249} - T_{501}$ (deg. F)
(X_1 =mean groove depth, and T=pavement temperature of the wetted pavement in these experiments).
a_i = the fitted constant, subject to errors.

It is understood that the calibrations are based on pairs of means of eight runs as the experimental unit.

Inclusion of D and T in the model will first be discussed. Groove depth and temperature were the two covariates in the analysis of variance. Their effect was found to be significant (Table 7), although similar for both tire types. It was therefore considered worthwhile to determine their effect on the calibration. Temperature difference is straightforward. Groove depth difference was initially computed on a percentage basis, since the two tires have different full groove depths. However, in subsequent computations, the simple difference in groove depths was used without affecting the significance of the results.

All equations were derived to give SNY as a function of SNX, plus some additional terms, if needed, as shown in Eq. 3. In many instances an inverse relation may be required, i.e. to obtain SNX as function of SNY. Similar equations could have been developed, but this additional work was not considered essential. Instead the given equations can be inverted to compute SNX from measured SNY. The error estimates will, strictly, no longer apply, but it is shown in Appendix C, that the inverted equations will provide almost the same unbiased prediction as would be accomplished through direct regression analyses.

In the preliminary analysis it was found that variability decreased with increasing speed (Table 9). The analysis of variance showed strong pavement and speed interactions. Also, it was surmised that the failure of "between mean" variances to be appreciably smaller than the "within" variances is due to pavement variability (Section 4.3). All this indicated the need to examine, in the calibration, various pavement and speed combinations, as well as each of the major factors.

4.4.1 Calibration at Different Speeds and Water Depths

The analysis of variance showed water depth to be a significant source of variation (Table 6), of about the same magnitude for both tire types. Hence, separate calibrations were made at each of the three speeds and two water depths (64 observations) as well as at both water depths combined. Table 10 shows the model and the resulting regression coefficients. Two questions arise as a result of this calibration procedure.

Table 10. Calibrations at different speeds and water depth.

MODEL: $SNY = a_0 + a_1\,SNX + a_2D + a_3T$

CONDITIONS		COEFFICIENTS			
SPEED MPH	WATER DEPTH, IN.	a_0	a_1	a_2	a_3
20	0.020	-3.35	1.06	-13.45	-0.09
	0.033	-2.21	1.05	21.82	-0.35
40	0.020	-1.06	0.98	7.63	-0.14
	0.033	-0.99	0.99	9.29	-0.01
60	0.020	-0.68	0.98	19.06	-0.01
	0.033	-0.24	0.97	22.32	0.01
20		-2.71	1.06	8.87	-0.21
40	BOTH	-1.09	0.99	8.30	-0.08
60		-0.46	0.98	21.11	-0.01

1. Do the calibrations differ at separate water depths?

2. Are the terms involving D and T necessary in the calibration?

An analysis of variance was made to answer the first question. Table 11, from OMNITAB regression outputs, focuses on the residual sum of squares. The first test performed for each pair of residuals, pertains to their homogeneity. The observed F-test is compared to the tabulated test value, $F_{60,60}$ (0.025) = 1.67, which is actually at the 0.05 level of significance, since either residual could have resulted in being larger. This test is applied at each speed and shows that the residual sum of squares for each water depth at a given speed do not differ significantly. This permits pooling the residuals for both water depths at each speed.

Table 11. Comparison of regression residuals between water depths.

CONDITION		RESULTS OF ANALYSIS				
SPEED MPH	WATER DEPTH, IN.	RESIDUAL S.S.	D.F.	MEAN S.S.	OBSERVED F	TEST F
20	0.020 0.033	311.0664 202.5464	60 60	5.1844 3.3758	1.54	1.67
	POOLED	513.6128	120	4.28		
	BOTH	540.5681	124	4.36		
	DIFFERENCE IN REGRESS.	26.9553	4	6.74	1.57	5.66
40	0.020 0.033	95.1013 72.9168	60 60	1.5850 1.2153	1.30	1.67
	POOLED	168.0181	120	1.40		
	BOTH	187.0227	124	1.51		
	DIFFERENCE IN REGRESS.	19.0046	4	4.75	3.39	5.66
60	0.020 0.033	157.6277 114.5673	60 60	2.6271 1.9095	1.38	1.67
	POOLED	272.1950	120	2.27		
	BOTH	273.4484	124	2.20		
	DIFFERENCE IN REGRESS.	1.3034	4	0.33	0.15	5.66

We now wish to answer the first question with respect to the similarity of the calibrations at each water depth. The "difference in regressions" with 4 d.f. is taken between the "pooled residuals" and the corresponding sum obtained from the model with combined water depths (128 observations). An F-test is again performed by dividing the "mean difference in regressions" by the "mean pooled sum of squares". It is found that all three differences in sets of regressions are not significant, since each of the resulting F values (1.57, 3.39 and 0.15 in Table 11) is less than the critical value ($F_{4,120}$ (0.05) =5.66). Thus, the calibration for each speed can be constructed using the data for both water depths (128 observations). This is consonant with the findings in Table 8 that the effect of water depth is the same for both tires.

To answer the second question, an analysis of variance was made to test the utility of including the D and T terms (Eq. 3) in the calibration. By examining the reduction in the residual sum of squares, it is found that, generally, including either term in the fitted model leads to a significantly smaller mean error variance. It should be noted that, although the inclusion of these two terms improves the fit of the data, the resulting corrections may indeed be minor, when compared to the contribution of the first two terms. Generally, calibration equations should be examined with respect to the precision of predictions (predictability) as well as to the quality of data fit. The addition of terms could lead to a greater variance in the prediction, even though the added terms improve the fit to the data (13). This aspect will be examined in a subsequent section, where comparison of different regression models will be made.

4.4.2 Calibrations Under Separate Speeds

Following the conclusions in Section 4.4.1, the combined data over both water depths were used in performing the following calibrations. Table 12 shows the regression equations at each of the three speeds, and gives the corresponding coefficients, for three separately fitted models. The first model involves all terms as given in Eq. 3, while the subsequent two models drop the T and D terms in turn.

Table 12. Calibrations at separate speeds.

CONDITIONS	COEFFICIENTS			
SPEED, MPH	MODEL: $SNY = Q_0 + a_1 SNX_1 + a_2 D + a_3 T$			
	a_0	a_1	a_2	a_3
20	-2.71	1.059	8.87	-0.214
40	-1.09	0.989	8.30	-0.082
60	-0.46	0.979	21.11	-0.006
	MODEL: $SNY = b_0 + b_1 SNX + b_1 D$			
	b_o	b_1	b_2	
20	-2.57	1.054	5.41	
40	-0.95	0.984	10.19	
60	-0.45	0.978	21.11	
	MODEL: $SNY = C_0 + C_1 SNX$			
	c_0	c_1		
20	-2.65	1.053		
40	-1.09	0.982		
60	-0.67	0.968		

The corresponding coefficients in the reduced equations ($SNY=c_0 + c_1$ SNX) for each speed are rather close to those in the full equations. This again indicates that D and T, although significant, may not be too important. In any event, these separate sets of equations will be examined as to their predictability. As might be expected, the calibration equations are characteristically dissimilar for the different speeds.

4.4.3. Calibrations Under Separate Pavements

Results of separate calibrations for each of the four pavements are given in Table 13. There is a rather strong consistency in the values of the corresponding coefficients of the reduced model equations to those computed for the full model involving SNX, D and T. These are rather strong indications that the simplest form among the equations ($SNY=c_0 + c_1$ SNX) may be most useful for predictability, in addition to the added convenience of fewer computations.

Table 13. Calibrations at individual pavements.

CONDITIONS	COEFFICIENTS			
PAVEMENT NO.	MODEL: $SNY_1 = a_0 + a_1\,SNX + a_2D + a_3T$			
	a_0	a_1	a_2	a_3
1	-4.36	1.087	33.03	-0.096
2	-0.41	0.942	11.86	-0.075
6	-2.84	1.083	18.77	-0.100
11	0.32	0.901	-3.80	-0.052
	MODEL: $SNY = b_0 + b_1SNX + b_2D$			
	b_0	b_1	b_2	
1	-4.77	1.096	34.86	
2	-0.38	0.942	11.42	
6	-3.04	1.088	20.08	
11	0.37	0.900	-3.48	
	MODEL: $SNY = c_0 + c_1SNX$			
	c_0	c_1		
1	-4.23	1.064		
2	-0.54	0.938		
6	-2.11	1.046		
11	0.37	0.904		

The sets of coefficients are quite different for the four pavements, but the differences for pavements 1 and 6, which exhibited similar skid resistance, are small. As similar observation can be made for pavements 2 and 11, which also had similar skid resistance.

4.4.4 Calibrations for Pavement and Speed Combinations

Because of the high PxV (pavement-speed) interaction in the analysis of variance (Section 4.2) it was concluded that calibration for various speed and pavement combinations should be examined. The resulting equations, for each pavement-speed combinations, were disappointing. Table 14 lists the coefficients for the full model. The results for the curtailed models are not presented, since their coefficients closely resemble the corresponding ones in the full model.

Table 14. Calibrations by pavement and speed.

MODEL: $SNY = a_0 + a_1 SNX + a_2 D + a_3 T$

CONDITIONS		COEFFICIENTS			
PAVEMENT NO.	SPEED MPH	a_0	a_1	a_2	a_3
1	20	25.15	0.50	8.38	-0.36
	40	22.64	0.41	-12.36	-0.07
	60	11.09	0.64	2.86	-0.01
2	20	6.87	0.68	8.03	-0.11
	40	3.74	0.68	-1.70	-0.11
	60	9.38	0.13	-14.04	0.01
6	20	-4.39	1.15	40.22	-0.43
	40	4.01	0.84	1.73	-0.04
	60	5.33	0.80	-4.93	-0.03
11	20	7.25	0.61	-11.93	-0.12
	40	8.03	0.45	-19.35	0.02
	60	6.23	0.45	-19.88	-0.01

It is seen that the slopes (coefficients a_1) are not anywhere near the value of 1.00 (as they have been in Tables 12 and 13), while the constant terms (the intercepts a_0) seem to exert undue influence. This may be explained by the fact that for any given pavement and speed combination the resulting set of skid numbers has a small range of values.

4.4.5 Calibrations Excluding the Constant Term

In the preceding sections (4.4.2 to 4.4.4) calibrations between the two tires were examined for individual speeds and pavements. It was found that the resulting equations, after omitting in turn the terms involving T and D, were similar to the corresponding full calibration models with regard to the SNX coefficients.

In this section calibrations will be considered from which the constant term a_0 has been omitted, thereby forcing the fitted line to pass through the origin. The argument in favor of this approach is that a closer approximation of the underlying physical laws may be achieved. If one tire measures zero skid resistance, so should the other tire. Conversely, if the region of interest does not include the origin, the constant terms should be retained if a full linear model is assumed.

However, the possibility of improving the predictability of the calibration, by omitting the constant term as well as the terms involving groove depths (D) and temperature (T) should be considered. Despite the fact that these were shown to significantly improve the fitted regressions, their inclusion in the equations can increase the variance of a predicted calibration (14). The improvement in prediction will be demonstrated in Section 4.5. In this section, the calibration equations without constant terms will be presented and compared to the preceeding corresponding equations that include this term.

The coefficients for the model by pavement and speed are listed in Table 15 and should be compared with those listed in Table 14. Despite the fact that each equation is based only on 32 points, representing a narrow range of SN values, it is seen that the coefficients a_1 range from 0.899 to 1.029, in consonance with the full model equations obtained for separate speeds and separate pavements (Tables 12 and 13). In any event, the improvement relative to the a_1 coefficients in Table 14 is clearly seen. The corresponding coefficients for the reduced models (with D and T deleted in turn) are not shown in Table 15, since they are quite similar to the ones listed.

Table 15. Calibrations by pavement and speed, excluding the constant term.

Model: $SYN = a'_1\ SNX + a'_2\ D + a'_3\ T$

CONDITIONS		COEFFICIENTS		
PAVEMENT NO.	SPEED MPH	a'_1	a'_2	a'_3
1	20	1.006	22.190	-0.389
	40	0.963	17.022	-0.066
	60	0.964	41.662	-0.007
2	20	0.929	3.159	-0.109
	40	0.903	3.046	-0.158
	60	0.899	16.360	0.152
6	20	1.029	32.788	-0.449
	40	0.968	8.597	-0.043
	60	0.986	7.516	-0.043
11	20	0.916	14.454	-0.106
	40	0.909	3.661	-0.048
	60	0.921	4.017	-0.015

The results of the calibration by pavements, without the constant term, are given in Table 16, for comparison with the corresponding coefficients in Table 13. It should be noted that all the SNX coefficients are now consistently smaller than unity. Otherwise the pattern is the same as before, namely the coefficients for pavements 1 and 6 are of similar magnitude, as are those of pavements 2 and 11. Also, the coefficients hardly change when the terms involving T and D are successively eliminated.

Table 16. Calibrations at individual pavements, excluding the constant term.

CONDITIONS	COEFFICIENTS		
PAVEMENT NO.	MODEL: $SNY_1 = a'_1\ SNX + a'_2\ D + a'_3\ T$		
	a'_1	a'_2	a'_3
1	0.986	29.62	-0.215
2	0.924	12.82	-0.023
6	0.997	14.29	-0.106
11	0.918	-3.84	-0.054
	MODEL: $SNY = b'_1\ SNX + b'_2\ D$		
	b'_1	b'_2	
1	0.985	31.64	
2	0.925	12.32	
6	0.996	15.35	
11	0.919	-3.51	
	MODEL: $SNY = c'_1\ SNX$		
	c'_1		
1	0.968		
2	0.912		
6	0.985		
11	0.923		

Results of the calibrations by speed without the constant term are listed in Table 17, for comparison with the corresponding coefficients in Table 12. All SNX coefficients are smaller than unity and smaller than the corresponding coefficients in Table 12. This is reasonable since all constant terms in Table 12 were negative. Otherwise, the same observations apply as were made with regard to the calibrations by pavements.

Table 17. Calibrations at separate speeds, excluding the constant terms.

CONDITIONS	COEFFICIENTS		
SPEED, MPH	MODEL: $SNY = a'_1\ SNX + a'_2\ D + a'_3\ T$		
	a'_1	a'_2	a'_3
20	0.991	15.07	-0.177
40	0.957	12.31	-0.060
60	0.964	22.71	-0.002
	MODEL: $SNY = b'_1\ SNX + b'_2\ D$		
	b'_1	b'_2	
20	0.991	11.89	
40	0.957	13.35	
60	0.964	22.73	
	MODEL: $SNY = c'_1\ SNX$		
	c'_1		
20	0.983		
40	0.947		
60	0.945		

Finally, before proceeding tc predictability, composite calitration equations, based on all 384 observations, with and without the constant terms, will be examined. Table 18 lists the six equations and the corresponding coefficients. It can be seen that the SNX coefficients are greater (less) than unity when including (excluding) the constant term. For the range of SN values observed with the E 501 tire, the constant term in the first three equations will dominate the expression so that the predicted SNY value (for tire E 249) will be less than the SNX value, in spite of the coefficients being greater than unity.

Table 18. Calibrations for composite data (384 observations).

MODEL SNY =	COEFFICIENTS			
$a_0 + a_1 SNX + a_2 D + a_3 T$	-1.49	1.018	13.32	-0.087
$b_0 + b_1 SNX + b_2 D$	-1.41	1.015	13.66	
$c_0 + c_1 SNX$	-1.59	1.011		
$a'_1 SNX + a'_2 D + a'_3 T$		0.977	18.52	-0.067
$b'_1 SNX + b'_2 D$		0.976	18.55	
$c'_1 SNX$		0.963		

However, by omitting the constant term, the resulting coefficients clearly show that the expected SN for tire E 249 is less than that for the E 501 tire. The question before us now is, which of the different regression equations to select. This will be treated in Section 4.5.

4.5 Predictability of the Calibration Models

The predictability of a calibration can be thought of as "the variance of a predicted response." The calibration models examined so far were of the general form

$$y = \sum_{0}^{k} a_i x_i \tag{4}$$

where y is the response, a_i (i=0,1,...k) are the computed coefficients with covariance matrix W, and x_i (i=0,1,....k) are independent variables.

For $x_0 = 1$ the above model contains a constant term (intercept), while for $x_0 = 0$ the model does not involve a constant term. For every regression equation as given in Eq. 4, a residual or error variance is also obtained, which we label S_e^2.

The above model may be used as a predictor at a given set of values $x' = (x_{0_1} x_1 \ldots x_k)$ or $(x_1, x_2 \ldots x_k)$. If the prediction is to be used for estimating the mean of the population corresponding to x', then the variance of the predicted response is estimated by

$$S_p^2 = x' W x \tag{5}$$

where x is the transpose of the row vector x'. However, if Eq. 4 is to be used to estimate the response to an individual new observation at x', then the predicted variance is estimated by

$$S_y^2 = x' Wx + S_e^2 \tag{6}$$

In practice Eq. 6 is used to compute the prediction variance and will serve here as the criteria of an equation's predictability. The criterion for selection of the appropriate prediction model has been the subject of considerable research in recent years. Some of the accepted ranking criteria are relatively complex and the required computations have to be done by a computer. Also, for any particular experiment, such criteria would not agree on the same order of rank. However, a useful and simple criteria that gives results similar to Eq. 6 is the "average estimated variance (AEV) criteria" (14). In effect, the AEV criterion tends to give higher preferred ranking to simpler models. However, besides the pediction variance, we also must consider any bias in the prediction which may be introduced by a particular model within the range of values of practical interest.

Eight categories of calibration data sets have been evaluated. The first category is based on all 384 data points, pooling the data over all three speeds, four pavements and all other variable test conditions. The next three categories are based each on 128 data points, i.e. separate for each of the three speeds. Four more categories are each based on 96 data points, i.e. separate for each of the four pavements. Each category is considered as an intercept and non-intercept model as shown in Table 19. The b and b' models have been omitted from some of the computations, but this does not affect the conclusions to be discussed presently.

Table 19. Prediction Models.

EQ'.	INTERCEPT MODEL	EQ'.	NON-INTERCEPT MODEL
a	$SNY = a_0 + a_1\ SNX + a_2\ D + a_3\ T$	a'	$SNY = a_1\ SNX + a_2\ D + a_3\ T$
b	$SNY = b_0 + b_1\ SNX + b_2\ D$	b'	$SNY = b_1\ SNX + b_2\ D$
c	$SNY = c_0 + c_1\ SNX$	c'	$SNY = c_1\ SNX$

Each model was examined at four SN values of 10, 30, 50, and 70. Models a, a', b, and b', which include differences in groove depth D and temperature T, were evaluated under the assumption that predictions are desired from tire E 501 to tire E 249, with all conditions remaining the same, i.e. D and T were assumed to be identically zero. If in some cases a user should have non-zero values for D or T, the composite models should be used, since the coefficients for D and T, being based on the largest sample, are most reliable. The results, including prediction variances and standard deviations are listed in Tables 20 to 27.

The results in Table 20 are based on the composite model over all speeds and pavements. This model may be used for any speed between 10 and 70 mph. It also may be used for any pavement type normally found on public highways.

Table 20. Predictions for composite data models.

NOMINAL SNX	PREDICTION MODEL a			PREDICTION MODEL a'		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	8.69	2.9412	1.72	9.77	3.2047	1.79
30	29.05	2.9255	1.71	29.30	3.2183	1.79
50	49.41	2.9570	1.72	48.83	3.2455	1.80
70	69.78	3.0447	1.75	68.37	3.2863	1.81
	PREDICTION MODEL b			PREDICTION MODEL b'		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10						
30						
50						
70						
	PREDICTION MODEL c			PREDICTION MODEL c'		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	8.52	3.0928	1.76	9.63	3.4171	1.85
30	28.74	3.0722	1.75	28.89	3.4243	1.85
50	48.96	3.0988	1.76	48.16	3.4387	1.85
70	69.18	3.1726	1.78	67.42	3.4603	1.86

It can be seen that the variances are lowest with model a and highest with model c'. However, the predicted skid numbers $SN\hat{Y}$ diverge at the low values for models a and c, but converge at the high values. The converse is true for models a' and c'. It appears, therefore, that, despite the somewhat smaller variances with the intercept models, the non-intercept models give more realistic predictions.

Tables 21 to 23 list the results for the models at defined speeds. These would be expected to give improved predictions at the speeds on which the particular models are based, namely 20, 40, and 60 mph. Each model should be valid for such pavements as discussed for Table 20.

Table 21. Predictions at 20 MPH.

NOMINAL	PREDICTION MODEL a			PREDICTION MODEL a′		
SNX	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10	7.87	4.5800	2.14	9.91	4.9757	2.23
30	29.05	4.4212	2.10	29.74	5.0213	2.24
50	50.22	4.5001	2.12	49.56	5.1125	2.26
70	71.40	4.8165	2.20	69.38	5.2493	2.29
	PREDICTION MODEL b			PREDICTION MODEL b′		
	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10	7.98	4.7694	2.18	9.91	5.0801	2.25
30	29.06	4.6059	2.15	29.72	5.1265	2.26
50	50.15	4.6865	2.17	49.53	5.2193	2.29
70	71.23	5.0110	2.24	69.34	5.3585	2.32
	PREDICTION MODEL c			PREDICTION MODEL c′		
	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10	7.88	4.7273	2.17	9.83	5.1126	2.26
30	28.95	4.5528	2.13	29.50	5.1374	2.27
50	50.01	4.6183	2.15	49.16	5.1870	2.28
70	71.07	4.9237	2.20	68.83	5.2614	2.29

Table 22. Predictions at 40 MPH.

NOMINAL SNX	PREDICTION MODEL a			PREDICTION MODEL a′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	8.80	1.5597	1.25	9.57	1.6467	1.28
30	28.58	1.5324	1.24	28.71	1.6691	1.29
50	48.36	1.5926	1.26	47.85	1.7139	1.31
70	68.15	1.7401	1.32	67.00	1.7811	1.34
	PREDICTION MODEL b			PREDICTION MODEL b′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	8.89	1.6194	1.27	9.56	1.6734	1.29
30	28.59	1.5932	1.26	28.70	1.6958	1.30
50	48.28	1.6549	1.29	47.84	1.7406	1.32
70	67.97	1.8047	1.34	66.97	1.8078	1.35
	PREDICTION MODEL c			PREDICTION MODEL c′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	8.72	1.6563	1.29	9.47	1.7569	1.32
30	28.35	1.6238	1.27	28.41	1.7697	1.33
50	47.98	1.6801	1.30	47.35	1.7953	1.34
70	67.61	1.8251	1.35	66.39	1.8337	1.35

Table 23. Predictions at 60 MPH.

NOMINAL SNX	PREDICTION MODEL a			PREDICTION MODEL a′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	9.33	2.2613	1.50	4.64	2.2265	1.49
30	28.91	2.2527	1.50	28.92	2.2681	1.51
50	48.49	2.3809	1.54	48.20	2.3513	1.53
70	68.06	2.6459	1.62	67.48	2.4761	1.57
	PREDICTION MODEL b			PREDICTION MODEL b′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	9.34	2.2430	1.50	9.64	2.2088	1.49
30	28.90	2.2341	1.50	28.92	2.2496	1.50
50	48.47	2.3540	1.53	48.20	2.3312	1.53
70	68.04	2.6027	1.61	67.48	2.4536	1.57
	PREDICTION MODEL c			PREDICTION MODEL c′		
	SN$\hat{Y}$	VAR.	S.D.	SN$\hat{Y}$	VAR.	S.D.
10	9.01	2.4348	1.56	9.45	2.4448	1.56
30	28.38	2.4104	1.55	28.34	2.4672	1.57
50	47.75	2.5196	1.59	47.23	2.5120	1.59
70	67.11	2.7624	1.66	66.13	2.5792	1.61

Comparison of the prediction variances in Table 21 with those in Tables 22 and 23 shows that, at 20 mph, the variances are about twice those at the two higher speeds. Similar differences were shown for the "within" variances in Table 9. In comparing the intercept and non-intercept models, the variances of the latter are seen to be generally greater than the corresponding intercept model at the two lower speeds, but smaller at 60 mph. However, the differences in variance between intercept and non-intercept models, at each speed, are generally small.

Tables 24 to 27 list the results for separate pavements. These models should only be used for a pavement which can be represented by one of the four pavements used in this correlation program. The prediction variances on pavement Nos. 2 and 11 are smaller, since the skid resistance of these pavements is lower than on pavements Nos. 1 and 6 (see also Table 3). With the intercept models the prediction variances are smallest in the range of skid resistance measured on the particular pavement (SN values between 30 and 50 on pavements 1 and 6, and SN values between 10 and 30 on pavements 2 and 11). With the non-intercept models, the prediction variances remain essentially constant over the 10 to 70 SN range of skid resistance.

Table 24. Predictions for pavement type 1 (Portland cement concrete).

NOMINAL	PREDICTION MODEL a			PREDICTION MODEL a′		
SNX	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10	6.51	4.3894	2.10	9.86	3.6780	1.92
30	28...5	3.5323	1.88	29.58	3.7140	1.93
50	49.99	3.4745	1.86	49.31	3.7860	1.95
70	71.73	4.2158	2.05	69.03	3.8940	1.97
	PREDICTION MODEL b			PREDICTION MODEL b′		
	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10						
30						
50						
70						
	PREDICTION MODEL c			PREDICTION MODEL c′		
	$SN\hat{Y}$	VAR.	S.D.	$SN\hat{Y}$	VAR.	S.D.
10	6.42	5.1652	2.27	9.68	4.2712	2.07
30	27.71	4.1705	2.04	29.04	4.2904	2.07
50	49.00	4.0429	2.01	48.40	4.3288	2.08
70	70.29	4.7826	2.19	67.76	4.3864	2.09

Table 25. Predictions for pavement type 2 (Jennite).

NOMINAL SNX	PREDICTION MODEL a			PREDICTION MODEL a′		
	SNY	VAR.	S.D.	SNY	VAR.	S.D.
10	9.02	2.1783	1.48	9.24	2.1009	1.45
30	27.87	2.2148	1.49	27.72	2.1769	1.48
50	46.71	2.7153	1.65	46.20	2.3289	1.53
70	65.56	3.6797	1.92	64.68	2.5569	1.60
	PREDICTION MODEL b			PREDICTION MODEL b′		
	SNY	VAR.	S.D.	SNY	VAR.	S.D.
10						
30						
50						
70						
	PREDICTION MODEL c			PREDICTION MODEL c′		
	SNY	VAR.	S.D.	SNY	VAR.	S.D.
10	9.84	2.2411	1.50	9.12	2.1821	1.48
30	27.60	2.2670	1.51	27.36	2.2277	1.49
50	46.35	2.7689	1.66	45.61	2.3189	1.52
70	65.11	3.7468	1.94	63.85	2.4557	1.57

Table 26. Predictions for pavement type 6 (silicious gravel).

NOMINAL SNX	PREDICTION MODEL a			PREDICTION MODEL a′		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	7.99	5.0549	2.29	9.97	3.6935	1.92
30	29.65	3.7147	1.93	29.91	3.7591	1.94
50	51.30	4.5969	2.14	49.84	3.8903	1.97
70	72.96	7.7015	2.78	69.78	4.0871	2.02
	PREDICTION MODEL b			PREDICTION MODEL b′		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10						
30						
50						
70						
	PREDICTION MODEL c			PREDICTION MODEL c′		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	8.35	5.2601	2.29	9.84	3.8439	1.96
30	29.28	3.8937	1.97	29.54	3.8719	1.97
50	50.21	4.5090	2.12	49.24	3.9279	1.98
70	71.13	7.1059	2.67	68.93	4.0119	2.00

Table 27. Predictions for pavement type 11 (Jennite-sand).

NOMINAL	PREDICTION MODEL a			PREDICTION MODEL a′		
SNX	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	9.33	1.3606	1.17	9.18	1.2836	1.13
30	27.36	1.4308	1.20	27.53	1.3396	1.16
50	45.39	2.1553	1.47	45.88	1.4516	1.21
70	63.41	3.5342	1.88	64.24	1.6196	1.27
	PREDICTION MODEL b			PREDICTION MODEL b′		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10						
30						
50						
70						
	PREDICTION MODEL c			PREDICTION MODEL c′		
	$\hat{SNY}$	VAR.	S.D.	$\hat{SNY}$	VAR.	S.D.
10	9.41	1.3564	1.17	9.23	1.2826	1.13
30	27.50	1.4033	1.19	27.70	1.3122	1.15
50	45.59	2.0814	1.44	46.17	1.3714	1.17
70	63.67	3.3906	1.84	64.64	1.4602	1.21

In assessing the predictability of the 8 categories considered above, the change in variance in going from model a to c or a' tc c' is generally small. Of the 64 cases computed, the variance increases 51 times and decreases 13 times. Thus models a and a' seem to be somewhat better predictors, than c and c' respectively.

The choice between the intercept model a and the non-intercept model a' is more difficult. The prediction variances of the former are slightly smaller, but this advantage must be judged against the greater dependence of these prediction variances on test conditions and against the additional computational effort. Based on these considerations, the non-intercept models are reccmmended over the intercept models.

5. ADDITIONAL ANALYSES

Some analyses were performed which, although not directly required for the test tire correlation, are of interest to the general subject of skid resistance.

5.1 Speed Dependence of Skid Resistance

Speed is known to have a significant effect on wet pavement skid resistance. This was confirmed by the results of the analysis of variance. Moreover, with the large amounts of available data a reliable functional relationship could be established. Similar to the approach in Ref. 11, second order regressions were constructed, separately for the two tires on each of the four pavements. Figure 6 shows the regression equations and the computed curves. Also listed are the computed skid resistance-speed gradients at 40 mph (G_{40}).

The coefficients of the first order terms are all negative, while those of the second order terms are positive. The plotted curves have been extrapolated over a range from 10 to 80 mph. It can be seen that the speed dependence, at the standard test speed of 40 mph, is quite strong, but becomes progressively weaker at 50 and 60 mph.

5.2 Pavement Texture and Skid Resistance-Speed Gradients

The computed gradients (-dSN/dV) at 40 mph are plotted in Fig. 7 versus the texture depth, as measured by the putty method (App. B). No relation between gradients and texture depth can be discerned. Similar disappointing results have been reported in a recent California study (15). A better fit is obtained, however, by using "percent gradients," i.e., the gradient at a given speed divided by the skid number at this speed (16). The four points for each of the two tires fall on a smooth, decreasing curve. The same ranking holds for percent gradients up to 50 mph, but is upset at 60 mph and above, i.e., in the flat portion of the curves (Fig. 6).

It can be concluded that, to obtain a good measure of skid resistance-speed gradients, the following two conditions must be met:

1. The measured SN at each of the speeds must be reliable, i.e. a sufficient number of replicate measurements must have been made.

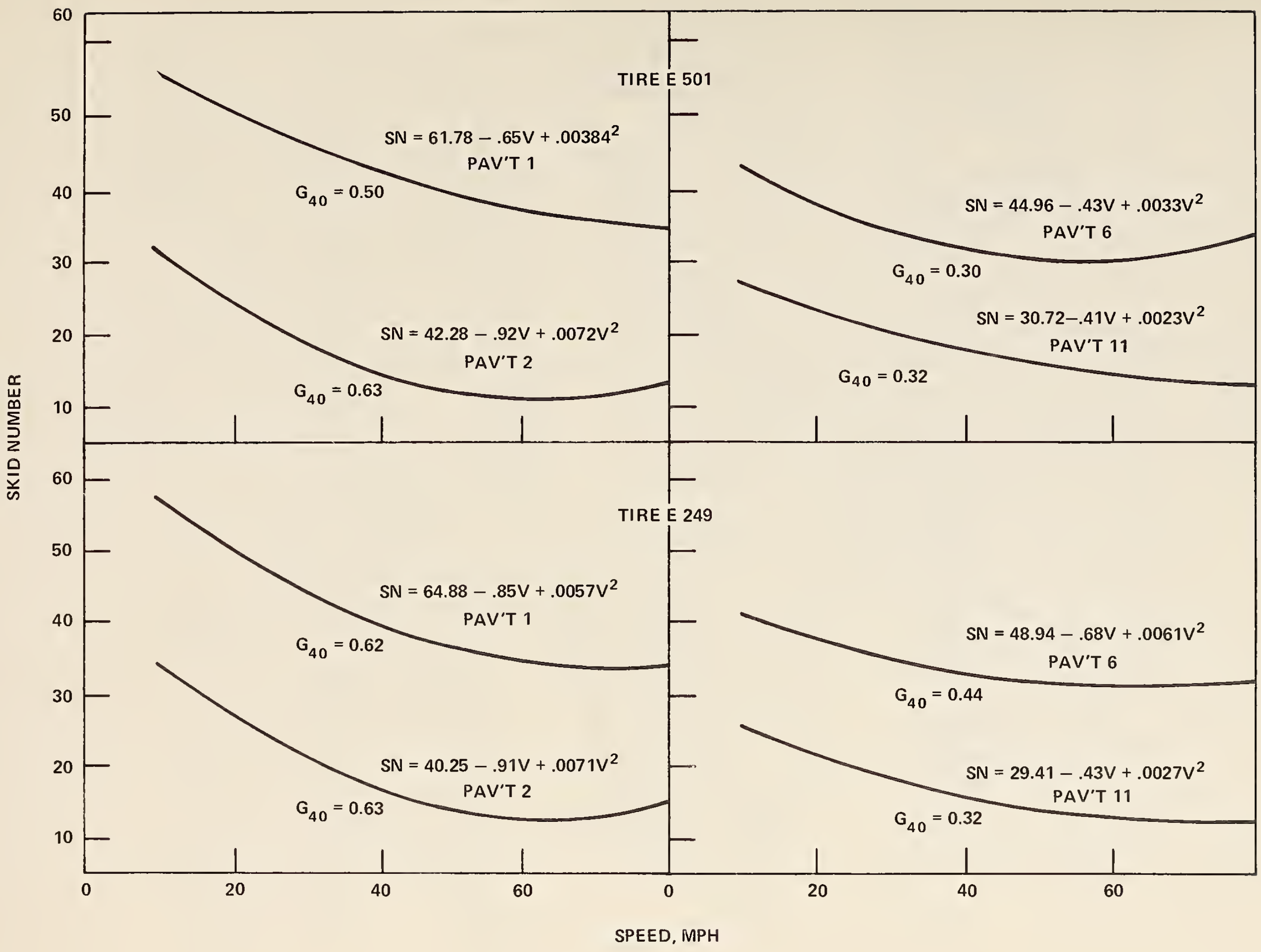


Figure 6. Second order regression equations; skid resistance versus speed.

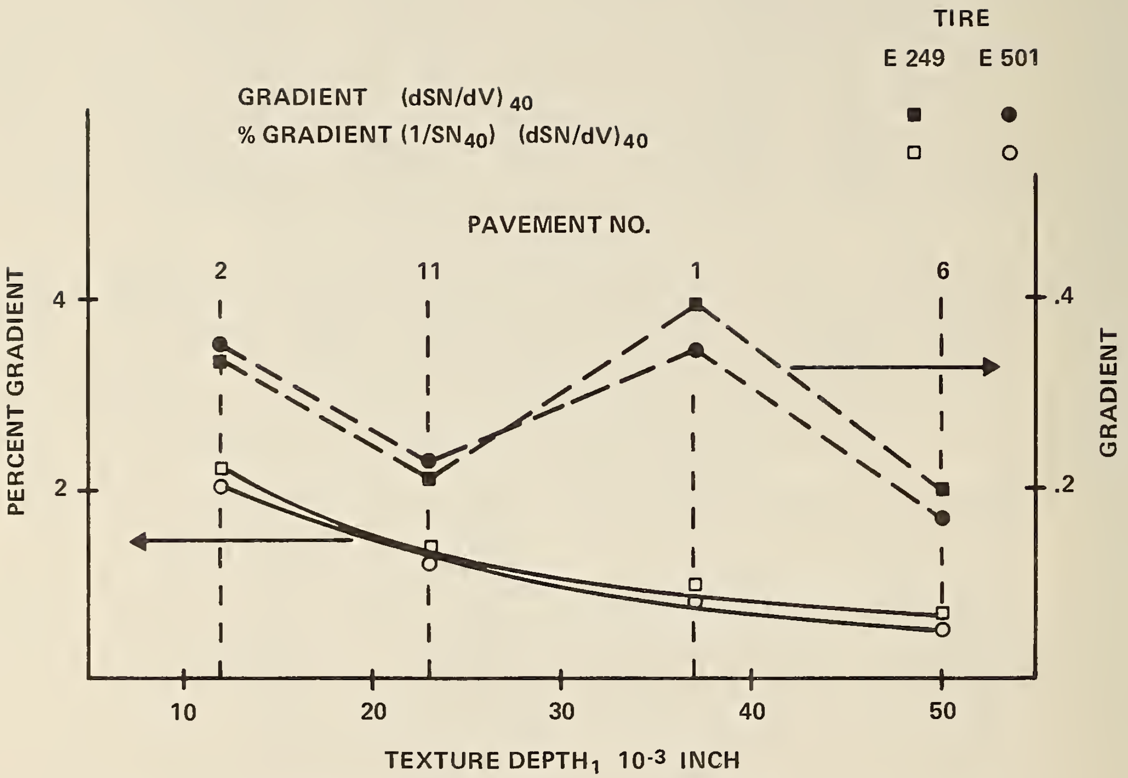


Figure 7. Skid resistance-speed gradients versus texture depth, by putty method.

2. Reliable skid resistance data at, at least, three distinct speeds, will allow to fit second order regression curves. From these, the gradients at any speed within the range of the data, can be computed.

It is recognized that in practice the number of measurements and speeds will be limited. In this case the value of a gradient is questionable. However, if needed, an estimate of the gradient may be obtained from measurements at two speeds, provided the data are reliable. The two speeds should obviously not be too far apart. On the other hand, because of the uncertainty associated with skid testing, measurements at two closely spaced speeds may miss the trend. A 10 mph speed difference is therefore considered a good compromise.

5.3. Effect of Temperature on Skid Resistance

It is generally accepted that skid resistance decreases with increasing temperature (17). This has been confirmed in the analysis of variance (Table 7). So far, however, a satisfactory skid resistance-temperature relationship, which would allow to apply consistent temperature corrections (11), has not been achieved. There are several possible explanations for this failure.

Firstly, while the temperature changes, other conditions do not remain constant either. This is especially true over longer periods of time. This problem complex is usually referred to as seasonal changes and requires urgent investigation. A good start has been made by including precipitation factors in the skid resistance-temperature relationship (18). Secondly, the temperature of the tire-pavement contact patch, which is probably the critical one, cannot be measured. Pavement temperature is the best approximation (19). During this test program wet and dry pavement, as well as ambient temperatures, were measured at least once every hour. Tire temperatures were measured immediately after completion of every 16 runs, between tire changes.

It was hoped that, using the large amount of data, it will be possible to establish reliable skid resistance-temperature relationships. Wet pavement temperatures were used in the regression analysis, because all tests were run on prewetted pavements. The temperature during the test program ranged from 40 to 110 degrees F, but this included all the different test conditions. The regression equations, over all tests, turned out to be inconclusive, probably because of the overriding effects of groove depths and waterfilm thickness.

Separate regressions for each data set resulted in more than half of the temperature coefficients being negative. However, the standard deviations of these coefficients were of the same order of magnitude as the coefficients themselves, so that no clear relationship can be determined at this time. The analysis merely confirmed the fact that, generally, increased temperature will cause a drop in skid resistance, but this is often obscured by the random variability. Table 28 lists the maximum temperature coefficients by tire type and pavement. We observe that the largest coefficients is

Table 28. Maximum temperature coefficients from skid resistance data at 40 MPH.

Model $SN = SN_0 + cT$

PAVEMENT	TIRE E 249		TIRE E 501	
NO	c	S.D.	c	S.D.
1	0.086	0.044	0.061	0.039
2	0.105	0.054	0.055	0.048
6	0.175	0.071	0.162	0.049
11	0.025	0.045	0.077	0.039

on pavement No. 6, which has the coarest texture (Fig. 7). Also, it appears that a 10 degree F increase in temperature will cause at the most, a drop of 1 SN, or about a 2 percent change in SN per 10 degree F change in temperature.

6. ADDITIONAL TESTS

6.1 Dry Pavement Tests

In skid resistance testing of pavements, dry tests are generally of no interest. However, dry pavement surfaces are often used for testing of tires and brakes. For this reason, a limited number of dry tests were included in both the field and laboratory test program.

The results in Table 29 show that, in all but one case, tire E 501 measured higher skid resistance than tire E 249.

Table 29. Skid resistance on dry surfaces.

FIELD TEST SURFACES	TIRE E 249	TIRE E 501
1	82.4	87.3
2	44.5	53.4
6	65.0	57.8
11	82.4	87.3
MEAN, ALL SURFACES	58.1	61.2
MEAN, WITHOUT SURFACE 6	55.8	62.3
LAB., TIRE CONDITION		
NEW	82.8	89.0
INTERMEDIATE	85.0	89.0
WORN	86.0	86.0
MEAN	84.6	88.0

Only for pavement No. 6 was the measured skid resistance higher with tire E 249 than with tire E 501. This pavement has been found to have the greatest variability (Table 5) and indeed the spread of the single skid data was large (22 SN). For this reason, data on pavement No. 6 are rather suspect. In light of all other data, including those from the laboratory, showing higher readings with tire E 501, we may conclude that tire E 501 will give higher readings than tire E 249, both on dry and on wet (Table 4) pavements. Data obtained by the Safety Research Laboratory (20) of NHTSA, confirm these findings. Two bituminous and one Jennite surface were tested by three skid testers. All tests were made at 40 mph and were completed in one day. All three testers measured higher skid numbers with the E 501 tire on all three surfaces. The mean difference on each surface was about 6 SN, which is approximately 7 percent on the bituminous surfaces and 11 percent on the Jennite surface.

Because of the limited data base, no statistical analysis was made of the dry test data. Mean values have been computed (Table 29) and it appears that skid resistance measured on dry pavements with tire E 501 may be between 5 and 10 percent higher than if measured with tire E 249.

6.2. Effect of Groove Width on Skid Resistance Measurement

The first production run of tire E 501 produced tires with narrower grooves than specified. The grooves were about 0.175 inches wide instead of 0.200 inches. This was recognized early in the program, but too late to postpone the tests. It was, however, decided to conduct a limited comparison program when tires with correct groove width become available. Such tests were held during May 1975, under the same conditions as the primary correlation program.

A summary of the data is given in Table 30, where FE 501 and SE 501 stand for first and second production run, respectively. The data are plotted in Fig. 8 and appear to show a slight difference, with the first production run giving somewhat higher readings. The data were subjected to statistical tests for significance of the differences, with negative results. This is in contrast to limited test results at GM Proving Grounds (21). There it was found that the narrower grooves gave somewhat lower readings at the higher speeds. These tests, however, were conducted at 0.050 inches water depth, which is 2.5 times the standard water depth.

Table 30. Summary of test data with first (FE 501) and second (SE 501) production E 501 tire.

PAVEMENT	EE 501							SE 501							
	C_1H_1			C_2H_2				C_1H_1			C_2H_2				
	I_1	I_2	(1)	I_1	I_2	(1)	(2)	I_1	I_2	(1)	I_1	I_2	(1)	(2)	V, MPH
2	31.2	26.7	29.0	25.3	27.2	26.3	27.7	28.7	24.6	26.7	27.1	25.5	26.3	26.5	20
11	28.8	25.1	27.0	22.7	24.9	23.8	25.4	28.3	22.3	25.3	24.2	22.9	23.6	24.5	
1	52.5	49.5	51.0	47.1	45.4	46.3	48.7	51.3	49.8	50.6	44.9	48.0	46.5	48.6	
6	39.2	35.7	37.5	38.5	39.1	33.8	38.3	36.9	36.5	36.7	37.0	36.2	36.6	36.7	
2	17.1	16.3	16.7	15.7	16.2	16.0	16.4	18.5	16.6	17.6	16.4	16.1	16.3	17.0	40
11	16.9	16.4	16.7	16.6	16.7	16.7	16.7	18.6	17.7	18.2	17.8	17.8	17.8	18.0	
1	41.7	39.8	40.8	36.8	35.2	36.0	38.4	41.8	41.1	41.5	31.6	32.8	32.2	36.9	
6	33.3	31.8	32.6	33.2	32.9	33.1	32.9	32.2	31.4	31.8	30.4	30.0	30.2	31.0	
2	12.1	12.3	12.2	10.4	11.3	10.9	11.6	13.4	11.2	12.3	9.9	9.5	9.7	11.0	60
11	14.8	14.7	14.8	12.3	13.6	13.0	13.9	15.3	13.5	14.4	11.6	12.6	12.1	13.3	
1	40.6	36.2	38.4	28.7	26.7	27.7	33.1	40.0	40.2	40.1	22.2	24.3	23.3	31.7	
6	32.1	31.6	31.9	30.3	29.7	30.0	31.0	33.4	32.1	32.8	27.3	27.3	27.3	30.1	

C_1 new tire

C_2 worn tire

H_1 0.022 inch water

H_2 0.033 inch water

(1) Mean of I_1 and I_2

(2) Mean of C_1H_1 and C_2H_2

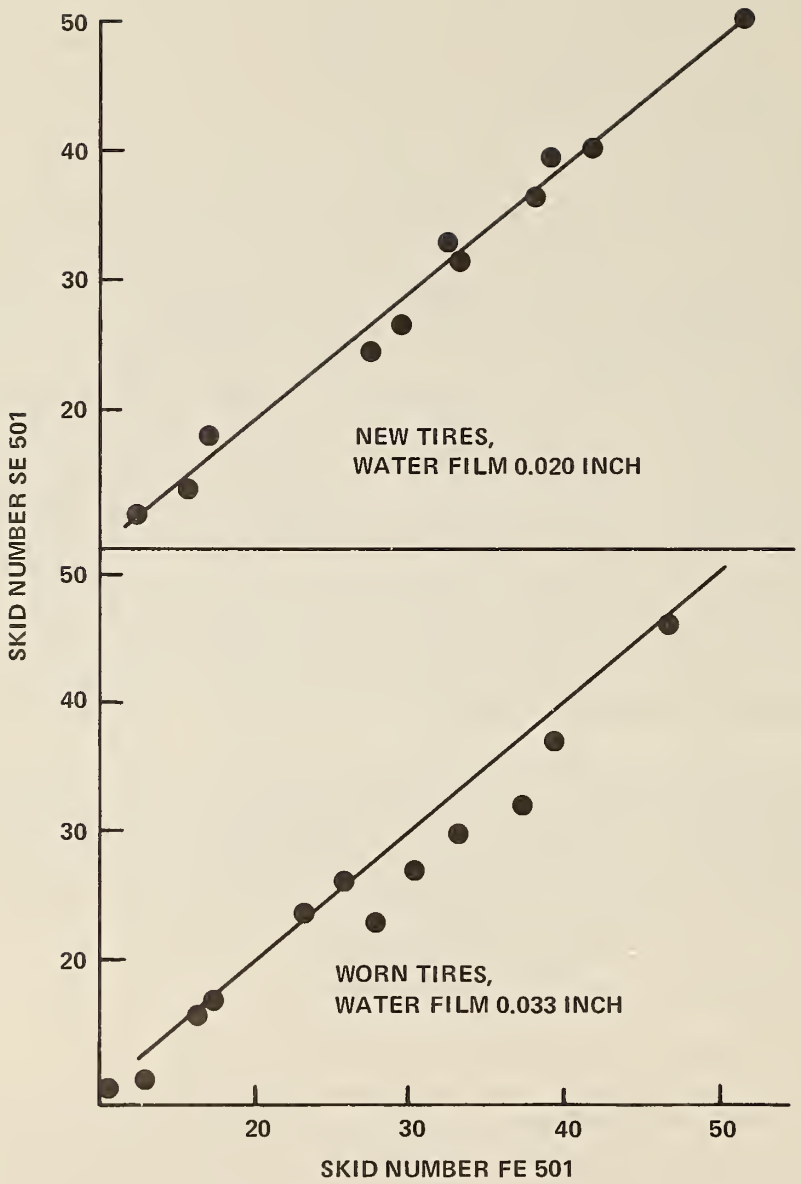


Figure 8. Skid numbers of second production run (SE 501) versus first production run (FE 501).

Based on the statistical analysis of our, albeit, limited data, we conclude that the groove width difference of 0.025 inches has no significant effect on skid resistance measurements, at the two water depths used in the test program. Consequently, the correlation equations, given in Table 1, are considered to be valid also for the tire with the correct groove width of 0.200 inches.

6.3 Laboratory Tests*

As mentioned in the Introduction, it was decided to conduct a limited laboratory test program in addition to the field tests with the skid tester. The high speed tire rest facility of CALSPAN Corp. (10) was used for these tests. It provides closer control of the test variables and allows for more convenient variation of some of the variables.

6.3.1. Test Conditions

The normal wheel load, which was constant in all field tests, was varied in four steps. Water film thickness was varied between zero (dry tests) and 0.060 inches in increments of 0.010 inches. Three inflation pressures, and the same three speeds as in the field tests, were used. Figure 9 gives the outline of the complete laboratory test program. It is to be noted that skid (or groove) depths are given in percent of total as well as of usable depth, i.e., the groove depth down to the wear indicator. The new tire (100 percent nominal) is designated 95 percent because of the slight wear in the 200 mile break-in. Two tires of each type were used, one in new and worn condition, the other in the intermediate condition. The order of runs is shown in Figure 10.

All tests were perfomed on the same surface which has a skid number of about 40 under standard test conditions. Tests on the wetted surface were repeated five times. On the dry surface, no repetitive runs were made to keep tread wear as low as possible. During every test, while wheel slip is increased from zero (free rolling) to 100 percent (locked wheel) a large number of closely spaced data were recorded. From these braking coefficients, F_x/F_z, and slip ratios, $(nR_e)/(168.07V)-1.00$, were computed where F_x is the longitudinal force, F_z is the vertical load, n is the wheel rotational speed in rpm, R_e is the loaded radius in inches, and V is the road speed in mph.

*Tables and figures in Section 6.3 have been prepared by CALSPAN Corporation.

RUN MATRIX

VERTICAL LOAD			938 lb	1085 lb						1232 lb	1380 lb
INFLATION PRESS.			24 psi	20 psi	24 psi			28 psi	32 psi	24 psi	24 psi
SPEED			40 mph	40 mph	20 mph	40 mph	60 mph	40 mph	40 mph	40 mph	40 mph
WATER DEPTH	SKID DEPTH OF TOTAL	OF USABLE									
0 mil	95% 70% 35%	100% 46% -18%				1,(2) 18,(19) 32,(33)					
10 mil	95% 70% 35%					16,(17) 21,(20) 44,(43)					
20 mil	95% 70% 35%		7,(3)	(4)	45,(46)	7,(3) 22,(23) 45,(46)	45,(46)	(5)	(6)	7,(3)	7,(3)
30 mil	95% 70% 35%					8,(9) 25,(24) 41,(42)					
40 mil	95% 70% 35%					11,(10) 26,(27) 40,(39)					
50 mil	95% 70% 35%					12,(13) 29,(28) 37,(38)					
60 mil	95% 70% 35%				35,36,(34)	15,(14) 30,(31) 35,36,(34)	35,36,(34)				

OPEN NUMBERS - STANDARD TEST TIRE ASTM E249 PARENTHESIZED NUMBERS - STANDARD TEST TIRE ASTM E501

Figure 9. Laboratory test matrix.

Run No.	E249 Tire No. 1	E249 Tire No. 2	E501 Tire No. 4	E501 Tire No. 5	Skid Depth
1	x				
2			x		
3			x		
4			x		↑
5			x		
6			x		
7	x				
8	x				95%
9			x		
10			x		
11	x				
12	x				↓
13			x		
14			x		
15	x				
16	x				
17			x		
18		x			↑
19				x	
20				x	
21		x			70%
22		x			
23				x	

Run No.	E249 Tire No. 1	E249 Tire No. 2	E501 Tire No. 4	E501 Tire No. 5	Skid Depth
24				x	
25		x			70% (cont)
26		x			
27				x	
28				x	
29		x			
30		x			↓
31				x	
32	x				
33			x		
34			x		↑
35	x				
36	x				
37	x				
38			x		
39			x		35%
40	x				
41	x				
42			x		
43			x		↓
44	x				
45	x				
46			x		

Figure 10. Test sequence and tire condition.

6.3.2 Test Results

The complete set of raw data for peak and slide braking coefficients are given in Appendix F. The slide braking coefficients or skid numbers for the two tires and their dependence on various parameters are discussed in this section.

Figure 11 shows that with the tires in new condition the skid resistance of the E 501 tire is consistently higher than that of the E 249 tire. Also, the skid resistance of both tires decreases weakly with water depth - about 1 percent per 0.01 in water depth. Figure 12 exhibits the same trends for 70 percent skid depth. At 35 percent skid depth (Fig. 13) the differences between the two tires tend to disappear. The data scatter and also the standard deviations of the means (indicated by the length of the bars) is large and make a meaningful data interpretation difficult.

From these data, the average difference and its standard deviation for a given range of water depth were computed, as shown in Table 31. The smallest standard deviation is obtained for unworn tires if the water depth is kept between 0.01 and 0.05 inches, and for tires with 70 percent groove depth, if it is kept between 0.01 and 0.04 inches. For tires with only 35 percent groove depth no reliable data can be expected at any water depth, since the standard deviations are as great or greater than the differences in skid numbers.

Table 32 lists mean standard deviations for the two tires at different water and groove depths. Similar to the findings in the field tests (Table 3), the standard deviations of the E 501 tire are only slightly greater than those of the E 249 tire. For a fully worn tire, however, this difference becomes significant, but this may improve with the increased groove width of the future production runs. Figure 14 shows the skid resistance of the new E 501 tire as a function of cold inflation pressure. At 1085 lb load, 40 mph, and 0.02 inch water depth, an increase of 1 psi decreases the skid number by about 0.35 percent.

Figure 15 shows the skid resistance of both tires in new condition as function of vertical load. At 24 psi and 0.02 inch water depth, an increase of 100 lb decreases the slide braking coefficient of the E 249 tire by about 1.1 percent, and of the E 501 tire by about 0.4 percent. Hence, the E 501 tire is apparently less sensitive to load variations than the E 249 tire.

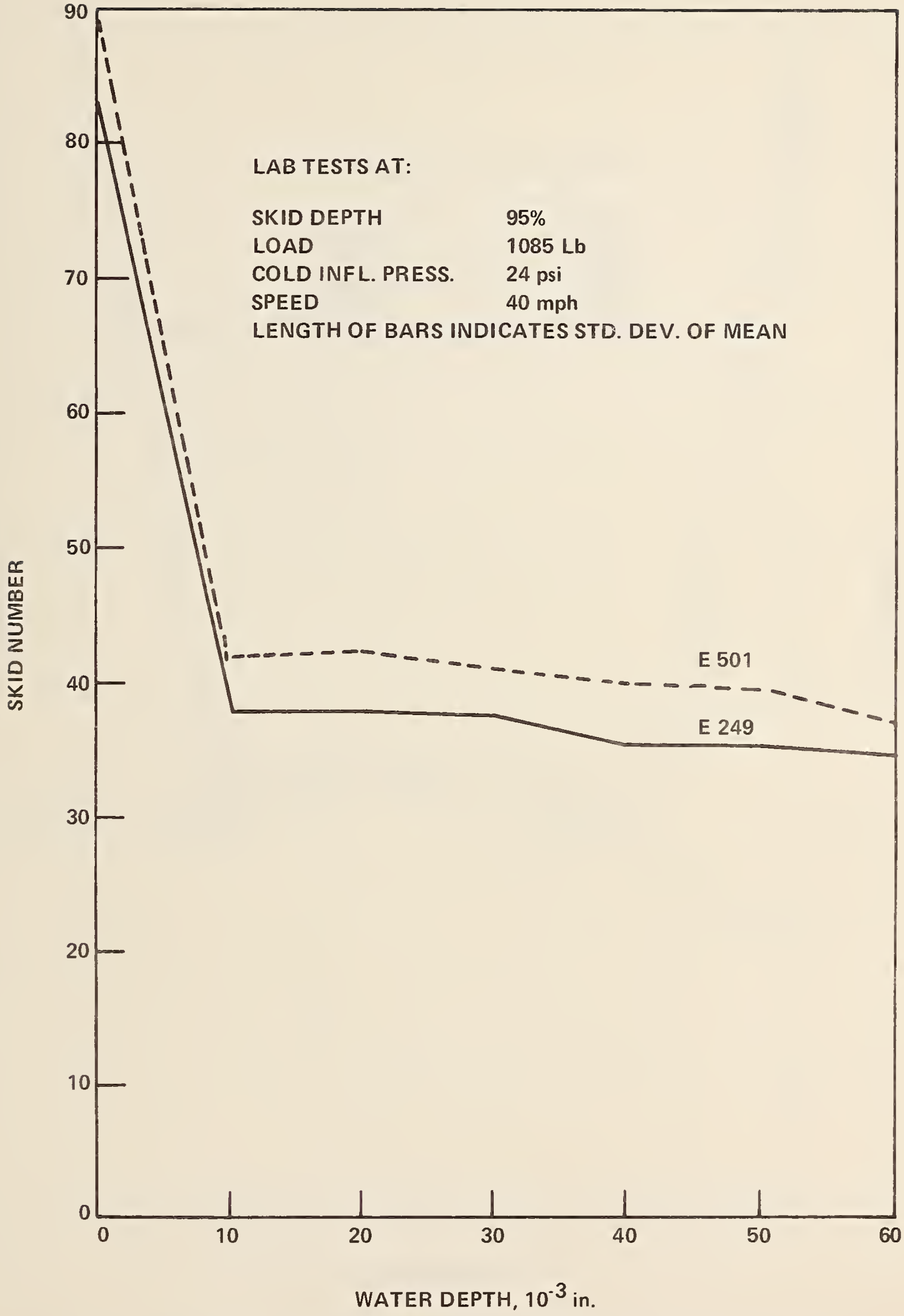


Figure 11. Laboratory skid resistance versus water depth, new tire condition.

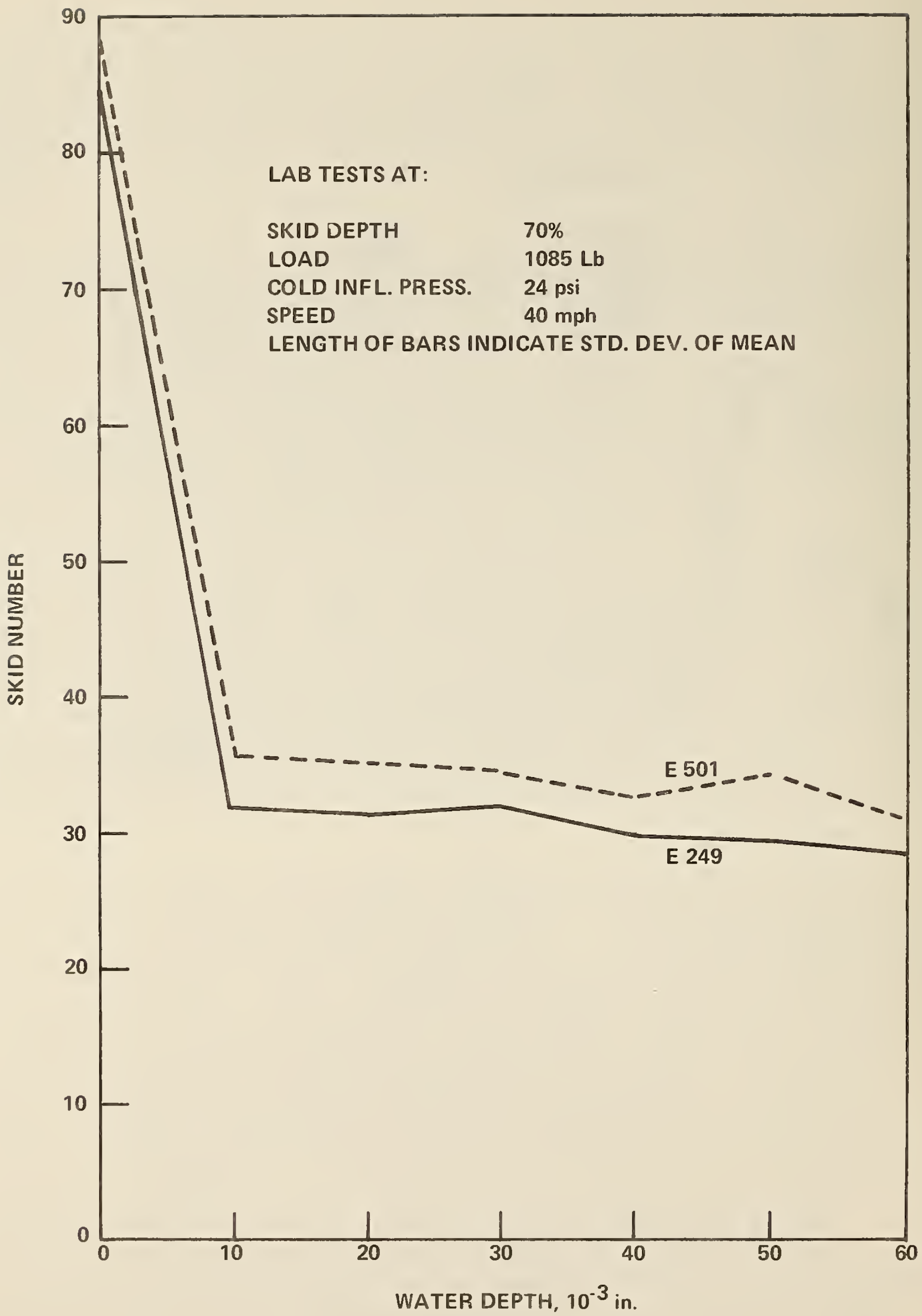


Figure 12. Laboratory skid resistance versus water depth, intermediate tire condition.

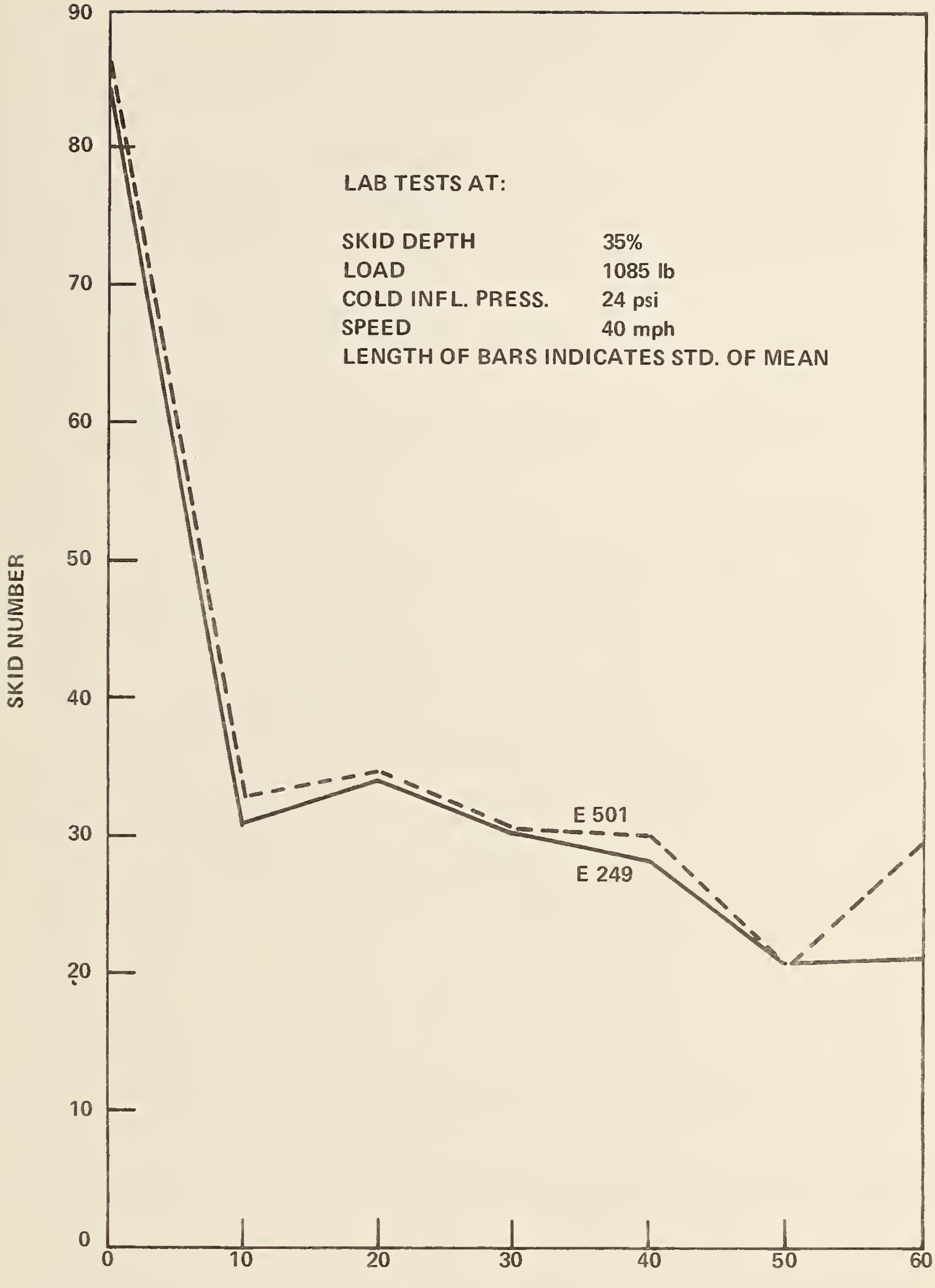


Figure 13. Laboratory skid resistance versus water depth, worn tire condition.

Table 31. Mean differences in skid numbers with standard deviations, at standard load, inflation pressure and speed.

GROOVE DEPTH PERCENT OF USABLE	TOTAL	WATER DEPTH, In. 0.01 - 0.04	0.01 - 0.05	0.01 - 0.06
100	95	4.2 (± 0.6)	4.2 (± 0.6)	3.9 (± 0.9)
45	70	3.1 (± 0.6)	3.4 (± 1.0)	3.2 (± 1.0)
-10	35	1.5 (± 1.4)	1.1 (± 2.2)	2.3 (± 3.7)

Table 32. Standard deviations of skid numbers, at standard load, inflation pressure and speed. Tire E 501 data in parentheses.

GROOVE DEPTH PERCENT OF USABLE	TOTAL	WATER DEPTH, In. 0.01 - 0.04	0.01 - 0.05	0.01 - 0.06
100	95	(0.39) 0.33	(0.37) 0.33	(0.39) 0.31
45	70	(0.33) 0.32	(0.37) 0.30	(0.34) 0.29
-10	35	(1.04) 0.42	(1.40) 0.64	(1.27) 0.84

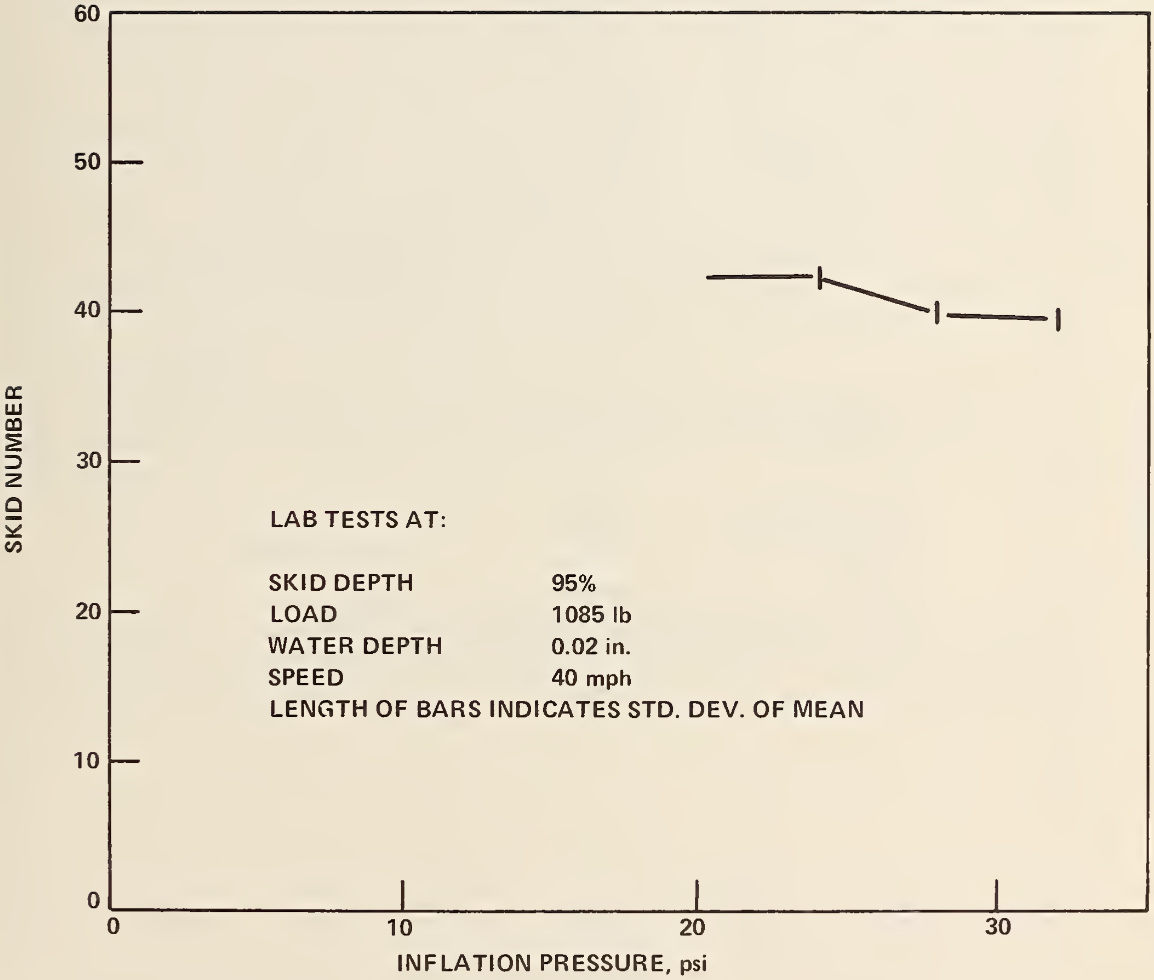


Figure 14. Laboratory skid resistance versus inflation pressure, tire E 501, new condition.

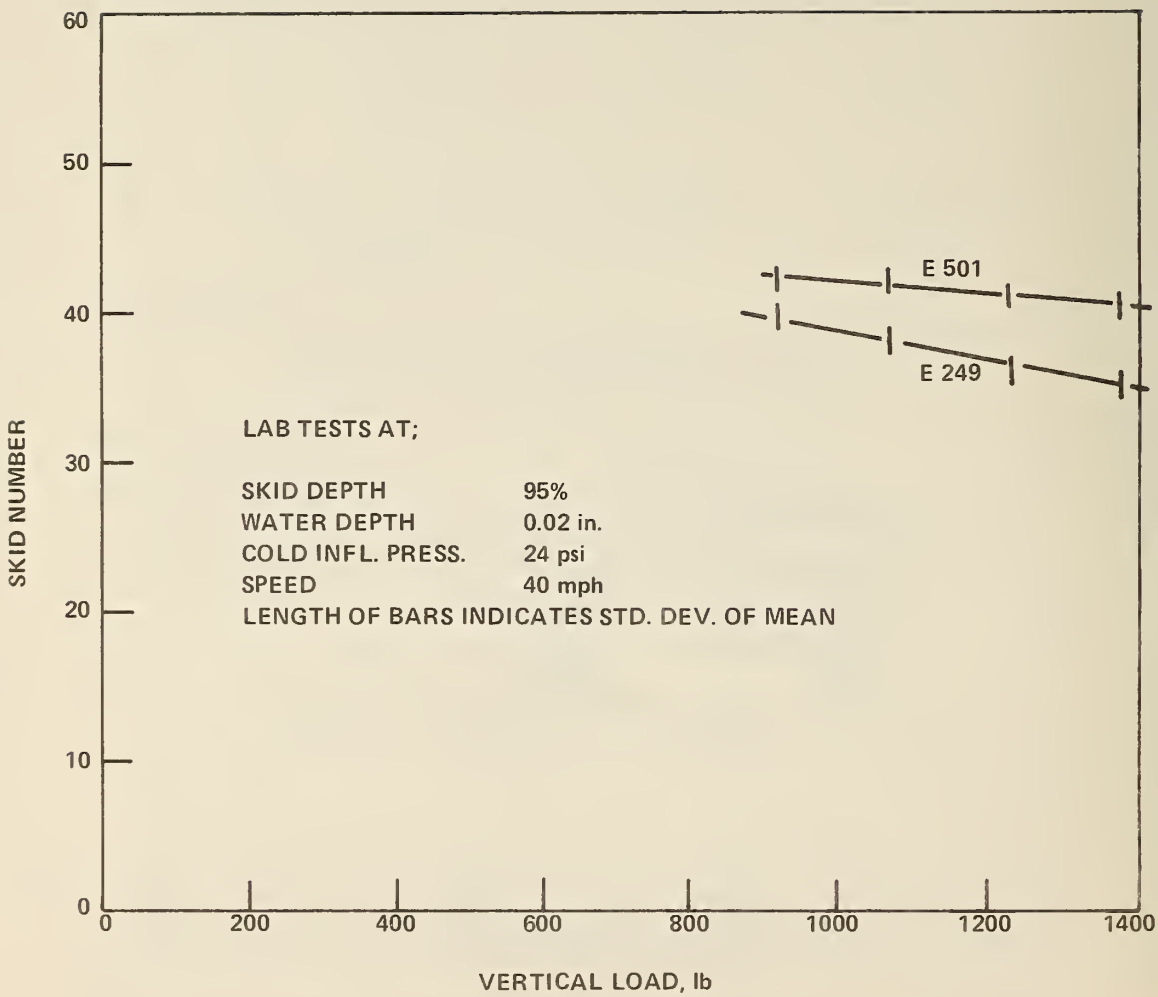


Figure 15. Laboratory skid resistance versus vertical load.

Figure 16 shows the effect of speed on the skid resistance of both tires with 35 percent groove depth (worst case). The decrease with speed is strong and suggests that skid tests require good speed control.

The effect of tire wear (measured by groove depth) can be seen in the next three figures. Figure 17 shows for both tires a drop of about 14 percent in SN. This is much greater than was found in the field tests (Fig. 5), but more importantly, the drop occurs while the usable groove depth decreases to about 50 percent, with no appreciable change afterwards. Figure 18 shows the difference in skid number between the two tires and the standard deviation of the difference. It appears that the difference decreases linearly with decreasing groove depth. Finally, Fig. 19 shows how the standard deviations vary with tire wear. They are somewhat higher in new condition, fairly constant over a wide range, but increase steeply at full wear.

The conclusions to be drawn from the laboratory tests are in general agreement with those from the field tests. A regression equation of the 40 mph data at all water depth resulted in

$$SN_{249} = 0.98\, SN_{501} - 2.51 \qquad (7)$$

which shows that for all practical purposes, tire E 501 will measure higher than tire E 249, with the difference being of the same order of magnitude as in the field tests.

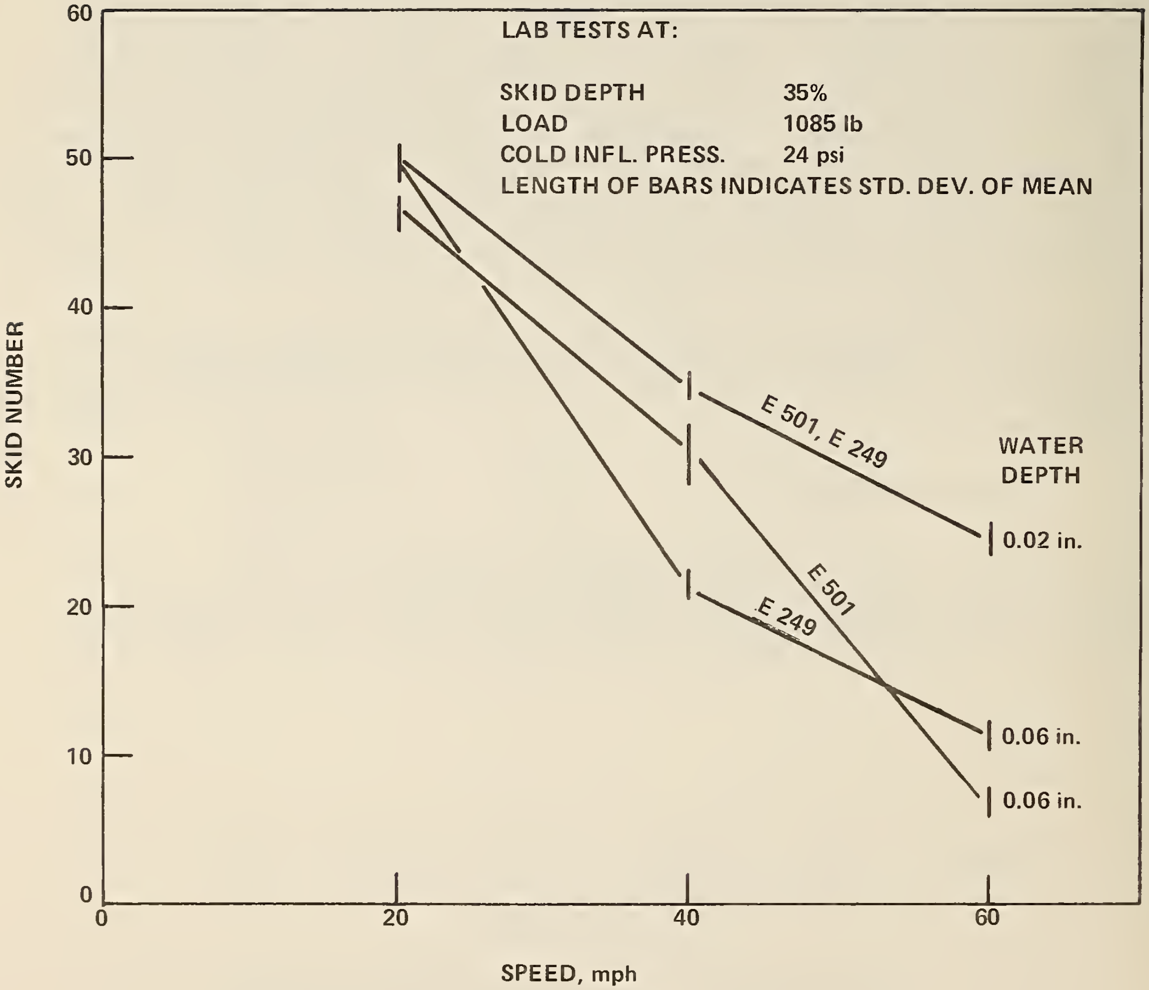


Figure 16. Laboratory skid resistance versus speed.

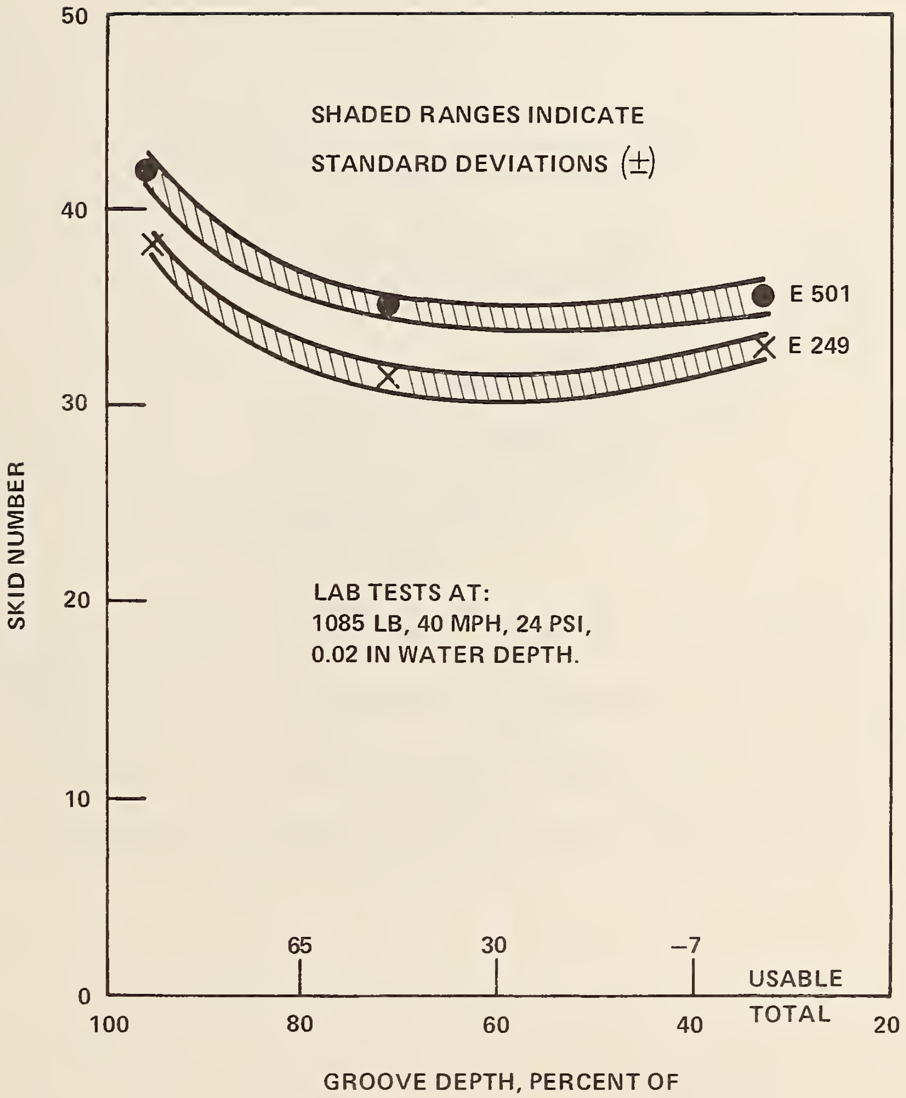


Figure 17. Skid number versus groove depth.

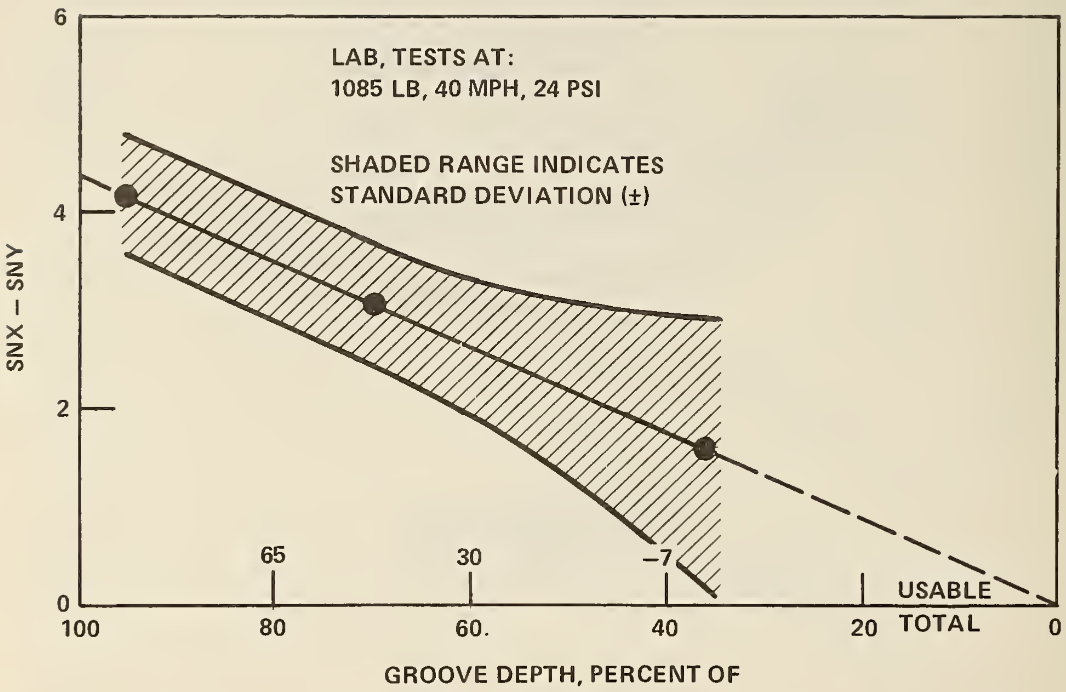


Figure 18. Skid number difference versus groove depth. Averaged over water depths between 0.01 and 0.04 inches.

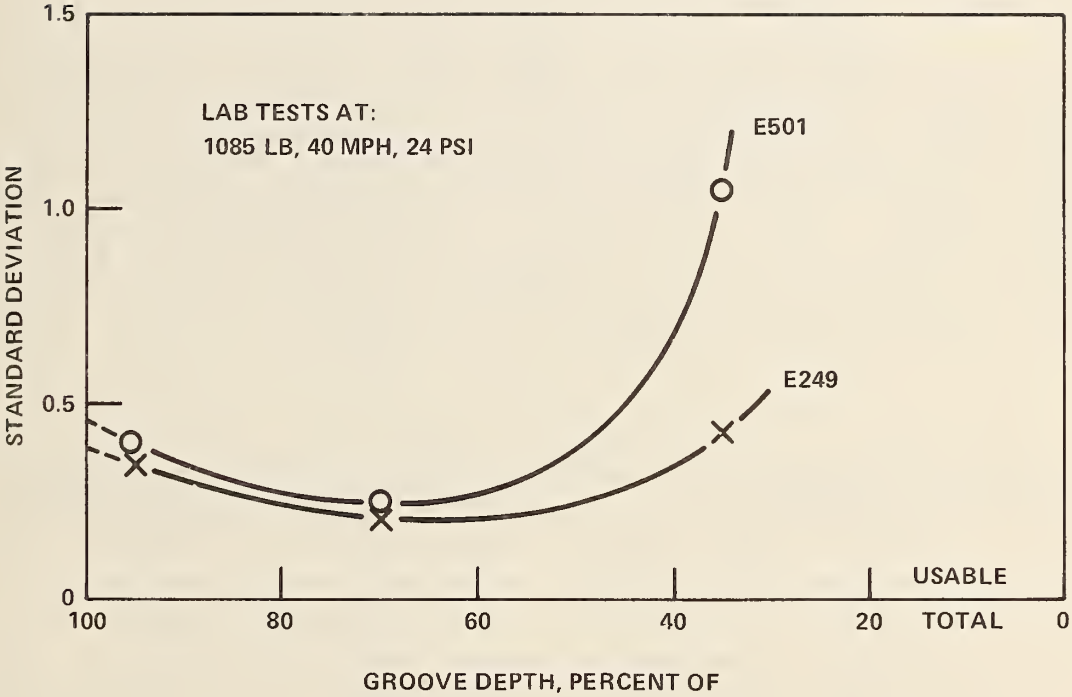


Figure 19. Standard deviations versus groove depth. Averaged over water depths between 0.01 and 0.04 inches.

7. CONCLUSIONS AND RECOMMENDATICNS

A large scale test program was conducted for establishing correlations between the newly standardized test tire (ASTM E 501) and the previous test tire (ASTM E 249). Both tire types were field tested with the same skid tester, on four pavements and at three speeds. In addition, high speed laboratory tests were run, which produced results in general agreement with the findings of the field tests.

The recommended correlation equations, based on multiple regression analyses of the field data are given in Table 33. Skid resistance measured with the E 501 tire is somewhat higher than, when measured under identical conditions, with tire E 249. The relatively large variability in skid testing may, however, cause reversals of this relaticnship, when only a small number of tests are made. Such occurrences must be accepted as part of the uncertainty in skid testing but do not invalidate the findings cf this program.

Table 33. Summary of correlation equation and associated variances.

EQ	PREDICTION: SNY =	VARIANCE (FOR D=T=O)	APPLICATION
A	0.977 SNX + 18.52 D - 0.067 T	$3.2030 + (0.0041\ SNX)^2$	GENERAL
B	0.991 SNX + 15.07 D - 0.177 T	$4.9700 + (0.0075\ SNX)^2$	20 MPH
C	0.957 SNX + 12.31 D - 0.006 T	$1.6434 + (0.0053\ SNX)^2$	40 MPH
D	0.964 SNX + 22.71 D - 0.002 T	$2.2213 + (0.0072\ SNX)^2$	60 MPH
E	0.986 SNX + 29.62 D - 0.125 T	$3.6735 + (0.0067\ SNX)^2$	PAV'T TYPE 1
F	0.924 SNX + 12.82 D - 0.073 T	$2.0914 + (0.0097\ SNX)^2$	PAV'T TYPE 2
G	0.997 SNX + 14.29 D - 0.106 T	$3.6853 + (0.0091\ SNX)^2$	PAV'T TYPE 6
H	0.918 SNX - 3.84 D - 0.054 T	$1.2766 + (0.0084\ SNX)^2$	PAV'T TYPE 11

All equations in Table 33 give the expected skid resistance of tire E 249 (SNY) as function of the skid resistance measured with tire E 501 (SNX), with two additional terms accounting for any difference in groove depth (D=groove depth of tire E 249 - groove depth of tire E 501, in inches) and difference in pavement temperature (T=temperature during testing with tire E 249 - temperature during testing with tire E 501, in deg. F). All other test conditions, such as speed, wheel load and water depth are the same for both tires. In most cases differences between groove depths and temperatures will either not be known or will be neglected. In this case the terms involving D and T drop out and there remains a simple relation between the skid numbers of the two tires, namely SNY=kSNX, where k represents the appropriate coefficient in Table 33.

Equations A to D have been obtained by averaging over the four pavements used in this program (App. B) and should therefore be valid for any pavement type normally found on public highways. Equation A may be used at any speed between 10 and 70 mph, while Equations B to D apply only at the indicated speeds. Equations E to H are valid only for pavements which are similar in every respect to the corresponding pavement in this program. These equations may also be used over the speed range 10 to 70 mph.

The coefficients for D and T in Table 33 vary over a wide range. These differences have no physical reality, but are caused by the uncertainty in the measurements. This is especially true for temperature measurements, where the coefficients vary by a factor of greater than 20. Whenever the terms involving D and T are to be included, Eq. A should be used, since the coefficients are based on a larger sample (384 data pairs of mean skid numbers) and have therefore more validity. However, for skid resistance data at the standard test speed of 40 mph Eq. C is recommended, provided the terms in D and T are neglected. The prediction variance at this speed has been found to be smaller than at the other test speeds and also smaller than with the composite model (Eq. A).

Table 33 also lists the prediction variances for each of the eight equations. The given values have been computed for the simple case of equal groove depth and equal temperature, i.e., D=0 and T=0, and are based on the sample size used in this correlation, namely eight skids. For a different sample of size n the first term in the variance equations should be multiplied by 8/n. Thus the prediction variance (or standard deviation which is the square root of the variance) increases as the number of skids per test site decreases.

The correlation between the two tires, over all conditions, is shown in Fig. 20. The computer prints a number whenever more than one point falls on the same coordinates (at the given resolution). The best fit line is

$$SNY = -1.49 + 1.018\ SNX \qquad \text{(Table 18)}$$

which is different from the recommended non-intercept prediction equation in Table 33 (Eq. A)

$$SNY = 0.977\ SNX$$

Dropping the constant term is justified because it simplifies the conversion and may improve the prediction (as discussed in App. C). In any case, the difference between the two equations is about 1 to 2 percent in the critical skid resistance range of 30 to 40 SN. This is much less than the percent standard deviation caused by pavement non-uniformity (Table 5).

Some tests were conducted on dry surfaces, both in the field and in the laboratory. These were limited tests and the data are insufficient for computing a correlation equation. The results show, however, that skid resistance measurements with the E 501 tire may be expected to be 5 to 10 percent higher than with the E 249 tire (Table 29).

Other important findings are:

- The "within" variances (variance among the eight repeat skids within each sample), as well as the "between" variances (variance among the mean skid numbers) are about the same for both tire types. The variance at 20 mph is, however, more than twice that at the two higher speeds (Table 9), therefore, low speed skid testing is not recommended, unless prevailing conditions make this necessary.

- The effect of increased water depth is about the same for both tires and may cause a drop of about 2 SN when doubling the standard water film thickness of 0.020 inches (Table 8).

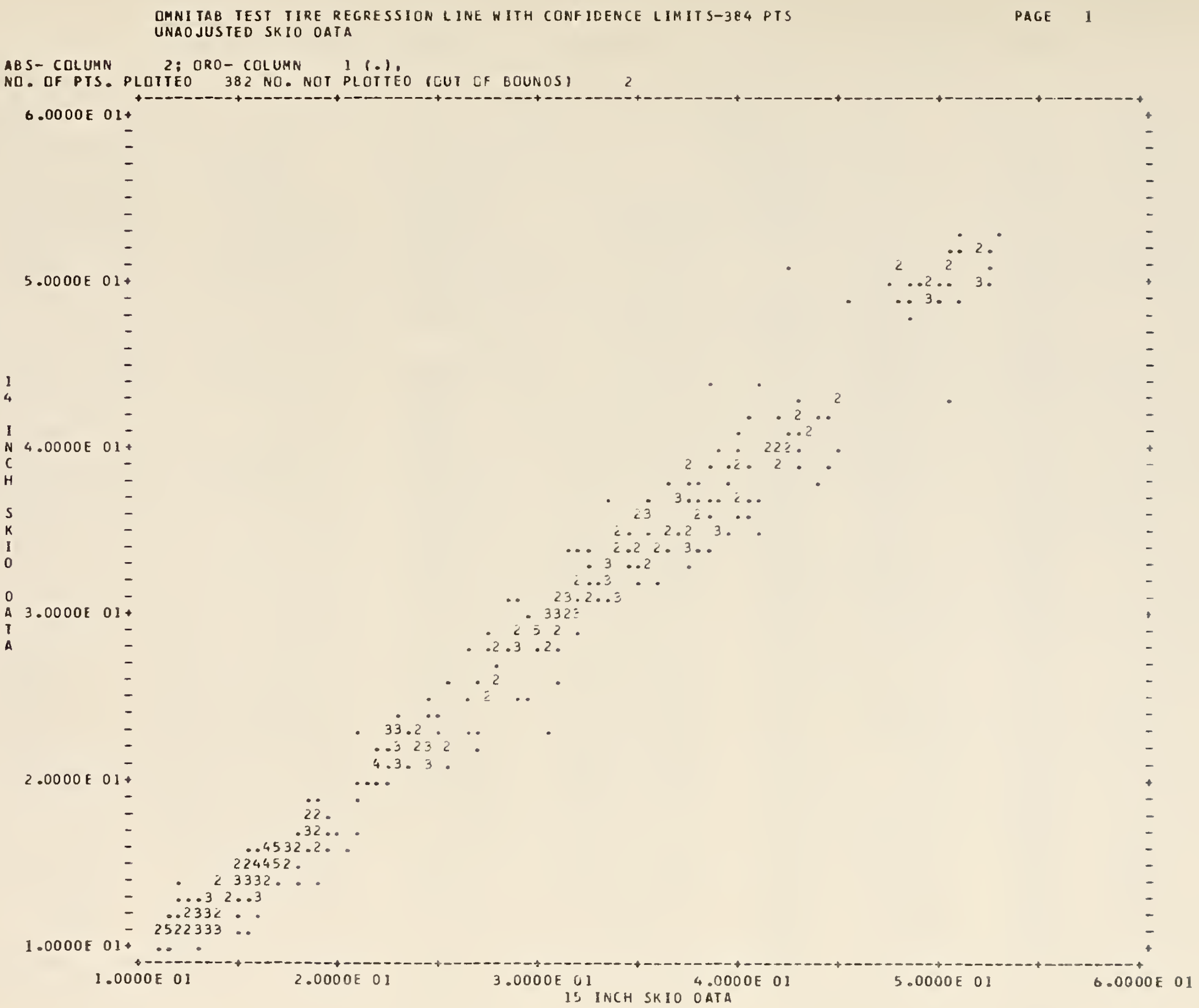


Figure 20. Mean skid number of tire E 249 (14 inch) versus mean skid number of tire E 501 (15 inch).

- Tire wear has a somewhat stronger effect on tire E 501 than on tire E 249 (Figs. 5 and 19). The drop in measured skid resistance is most pronounced during the initial wear (Fig. 17). The difference in wear effects between the two tires may vanish when the groove width of tire E 501 is corrected to meet the specifications. This groove width was, in the first production run, 0.175 inches instead of 0.200 inches. This has now been corrected. A brief test program was conducted to determine the effect of this change. Under the prevailing test conditions no systematic difference, as result of the different groove widths, could be found (Fig. 8).

- The effect of temperature on skid resistance is shown in Table 28. For a temperature increase of 10 deg. F a decrease in SN of at most 2 percent may be expected. It must be emphasized, however, that temperature effects are frequently submerged in other effects and, at present, no reliable correction method is known.

- Based on an analysis of four replications with different tires, all other conditions held constant, the conclusion is that tires, of the same type and same production run, do not differ significantly with respect to skid resistance measurement.

- The decrease of skid resistance with speed depends on the pavement macro-texture. Good correlation can be obtained between macro-texture and percent gradients, i.e., the skid resistance-speed gradient divided by the skid resistance at the same speed (Fig. 7).

Generally, both tires respond similarly to changing test conditions, so that skid testing with the new tire (E 501) is not expected to present more problems than were experienced with tire E 249. This statement does not, however, apply to tire wear, which will have to be judged from experience.

In summary, the equations given in the left column of Table 33 may be used to relate skid resistance measurements taken with one tire type to those of the other tire type. The corresponding variances are given for SNX, i.e., when skid resistance is measured with the new test tire. If, however, SNX is to be computed from a measured SNY, the latter can be used in the variance equations, without introducing significant errors (App. C).

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APPENDIX A

PROSPECTUS

TEST TIRE CORRELATION
(ASTM Designations E 249 and E 501)

INTRODUCTION

Production of the standard test tire E 249 by General Tire will be discontinued in the near future. A new test tire has been approved by ASTM Committee E 17 under designation E 501. This tire will be supplied by B. F. Goodrich Co. under Stock No. 398-571 and is available at an approximate unit cost of $60.00.

The E 501 tire differs in several respects from the E 249 tire. Physical specifications of interest are listed below.

	ASTM E 249	ASTM E 501
Size	7.50-14	G78-15
Rim	14x5J	15x6JJ
Tread width, inch	4.65	5.85
Ribs	5	7
Grooves (0.2 inch wide)	4	6
Groove depth, inch	0.350	0.358
Min. groove depth (wear indicator)	0.150	0.165
Inflation pressure, psi	24	24
Test load, lb.	1085	1085
Construction	bias-ply	bias-belted

STATEMENT OF WORK

CONTRACT OBJECTIVES

To determine correlation between the new (E 501) and present (E 249) ASTM Pavement Test Tires under specified conditions.

SCOPE OF WORK

A prescribed number of tires of each type will be tested under prescribed conditions on several selected pavement surfaces. Additional tests will be run on a high speed laboratory tire test facility.

DELINEATION OF TASKS

A. FIELD TESTS

Task 1. Select or prepare four pavement surfaces to span a skid resistance range of 20 to 60 SN approximately and be representative of surface textures found on real roads.

Task 2. Purchase 25 tires of each type and from the same production batch. Mount on wheels of correct size, balance and run according to ASTM Method E 274-70

Task 3. Prepare data sheets as per attached sample (Attachment 1).

Task 4. Conduct one day of exploratory tests, according to the procedure given in Attachment 2. Use the same tires in both series.

Task 5. Evaluate all phases of the exploratory tests of Task 4 in order to determine if the test program can be conducted as planned. If needed, recommend modifications of the procedures and submit to the contract manager for approval.

Task 6. Conduct the test program according to the procedure (Attachment 3).

Task 7. Evaluate the test records and submit the raw data and all other information relevant to the test program to the contract manager.

B. LABORATORY TESTS (on CALSPAN TIRF facility)

Task 1. Run tests on both types of tire as per attached procedure (Attachment 4). Use tires from same batch as in the field tests.

Task 2. Submit tabulated test results and plot friction data versus all controlled variables.

TECHNICAL GUIDELINES

Use same equipment, operators, and procedure throughout the test program. Omit all invalid test data and run substitute tests when possible. A test may be invalidated only if some clearly recognizable mishap has occurred during the test.

Experimental design parameters are listed and discussed below:

a. <u>Fixed Variates</u>

1.	Tire types:	T_1 (E 249), T_2 (E 501)
2.	State of tire wear:	C_1 (new), C_2 (worn)
3.	Weaterfilm depth (inch):	H_1 (0.20), H_2 (0.033)
4.	Pavement surface group:	P_1, P_2 , and P_3, P_4
5.	Surfaces within group:	P_1, P_2, P_3, P_4
6.	Period of day:	I_1 (morning), I_2 (afternoon)
7.	Speed (mph):	V_1 (20), V_2 (40), V_3 (60)
8.	Inflation pressure:	24 psi cold
9.	Wheel load:	1080 lb.

b. <u>Covariates</u>

1.	Mean groove depth of tires	x_1
2.	Wet pavement temperature	x_2
3.	Order of skids in any test	x_3

1. Tires: Twenty-five tires of each type will be purchased, from the same production batch. The tires shall be prepared according to ASTM Method E 274.

2. State of tire wear: Field tests will be made at two states of wear, new and worn. A tire will be considered "new" when worn less than 1/16 inch, and "worn" when groove depth is between wear indicator and 1/16 inch below. Groove depth is the mean of six evenly spaced measurements around the tire. All measurements on a tire must be with 0.1 inch of each other, otherwise the tire shall be rejected. Mean groove depth over all grooves shall be reported. For the worn stale tire treads will be cut or ground and run in again. Precision of measurements shall be 0.01 inch or better.

3. Waterfilm thickness: Nominal waterfilm thickness in the field tests shall be 0.020 inch as per ASTM Method E 274 and 0.033 inch. If possible, actual waterfilm thickness shall be measured and reported together with method of measurement. Skid resistance shall, however, be reported as function of nominal film thickness. Waterfilm thickness in the laboratory shall be according to Attachment 4.

4. Pavement surface group: The four surfaces shall be divided into two groups, selected to provided convenient test arrangement.

5. Surfaces within group: The two surfaces within each group shall be tested in the order given in Table A-2 (Attachment 3). Ten consecutive runs shall be made on each surface, including two prewetting runs without locking the test wheel. Select a lateral position on the surface of greatest possible uniformity and with no depressions where water may tend to accumulate. Maintain the same lateral position through all tests.

6. Period of day: Two test series shall be run per day at times to give greatest possible temperature difference.

7. Speed: Tests shall be run at three speeds in the tire-speed sequence given in Table A-1 (Attachment 3).

8. Inflation pressure: Inflation pressure shall be set at ambient temperature to 24 psi and checked after completion of test sequence. Inflation pressure in laboratory tests shall be varied according to Attachment 4.

9. Wheel load: Static wheel load shall be constant in all field tests. Load in laboratory tests shall be varied according to Attachment 4.

The three covariates will be used in the data analysis. Pavement and air temperatures shall be recorded at no more than one hour intervals. Tire temperature shall be taken at end of each test sequence. Temperature sensors and method of measurement shall be reported. Skid tests shall be evaluated and listed in order of runs under Numbers 1 to 8 on data sheet (Attachment 1). Mean tire groove depth shall be recorded as discussed under "State of wear."

Appendix A
Attachment 1
Page 1

FHWA - TTI
CENTRAL FTC

ASTM 14" & 15"
TIRE CORRELATION

FIELD DATA SHEET
TEST TIRE CORRELATION

Date:________________ Test Series No:________________

Time of Start:________________ of Finish:________________

Humidity:__________Ambient Temp. at Start:______, at Finish:______°F

Nominal Water Film Thickness 0.020 inch:______0.033 inch:______

Tire Condition C1 ___ C2 ___

Tire E249 Serial No.:________________
E501 Serial No.:________________
E249(1) Serial No.:________________
E501(1) Serial No.:________________

Condition	Test Sequence 1	2	3	4
Surface 1				
Surface 2				
Tire				
Speed				
Tire Temp.(2)				
Infl. Press(2)				
Mean Groove Depth(3)				
Tire Condition(4)				
Start Time				
End Time				
Comments(5)				

(1) Replacement tires, if needed
(2) Immediately after completion of sequence (tire pressure at mounting to be 24 psi).
(3) After completion of sequence
(4) G-Good, ER-Groove Depths Exceed Range, S-Shredded, B-Blistered
(5) Give reason for discarding tires, note repeated lockup in same position, etc.

Appendix A
Attachment 1
Page 2

FHWA - TTI
CENTRAL FTC

ASTM 14" & 15"
TIRE CORRELATION

DATA REDUCTION SHEET
TEST TIRE CORRELATION
SKID NUMBERS

Test Series No.: ________
Date: ________ AM ___ PM ___

Tire												
Surface												
Speed												
Group(1)												
Runs 1												
2												
3												
4												
5												
6												
7												
8												
Mean												
Variance												
St. Dev.												
Groove Depth, in.												
Pavement Temp. °F												

(1) Group is coded by letters A to L

FHWA - TTI
CENTRAL FTC

ASTM 14" & 15"
TIRE CORRELATION

TEST TIRE CORRELATION
HOURLY TEMPERATURE RECORD

Date: ____________

	Surface	Temperature 2		11		1		6			
	Time	D	W	D	W	D	W	D	W	Air	Humidity
AM											
PM											

	AM	PM
Test Series No.		
Sunny		
Cloudy		
Rain		
Wind, mph		

Comments:

D-Dry, W-Wet

Notes:
1. Pavement temperatures measured with ____________
2. Air temperatures measured with ____________
3. Measure temperatures at beginning and end of test series and at intervals of not more than 1 hour.

FHWA - TTI
CENTRAL FTC

ASTM 14" & 15"
TIRE CORRELATION

TREAD DEPTH MEASUREMENTS

Date:____________________ Tire No.:____________________

Test Day:____________________ Period:____________________

Test Sequence 1

Average____________________

Test Sequence 2

Average____________________

Test Sequence 3

Average____________________

Test Sequence 4

Average____________________

PROCEDURE FOR EXPLORATORY TESTS

Attachment 2

For definitions, see Attachment 3.

	First Series I_1	Second Series I_2
Tire Type	T_1, T_2	T_1, T_2
State of tire wear	C	C
Waterfilm depth	H	H
Pavement surface group m = 1,2; n = 3, 4	P_m, P_n	P_m, P_n
Speed	V_1, V_2, V_3	V_1, V_2, V_3

Use test plan as per Item B2 of Attachment 3. Repeat same for both series. In each surface group test one surface "prewetted" (as described in Item 5, Attachment 3) and one surface "dry," by lateral translation of successive runs.

FIELD TEST PROCEDURE

Attachment 3
Page 1

A. Definitions

Run: Individual pass over one test surface (either prewetting pass with rolling wheel or test pass with locked wheel).

Set: Ten consecutive runs at one speed over one surface and fixed test conditions.

Sequence: Two consecutive sets for one tire type over two surfaces, belonging to one pavement surface group.

Series: Three sequences for each tire type, at three speeds, at a given waterfilm depth H, state of tire wear C and period of day I, for a total of six sequences (three for each tire type).

Tire-speed sequence: In order to minimize variations in test conditions, the two tire types will be tested back to back, i.e., one after the other at the same conditions. With three speeds and two tire types there are 12 possible combinations, listed in Table A-1.

Table A-1-Tire-Speed Sequences

Sequence	T, V	T, V	T, V	T, V	T, V	T, V
1	1, 1	2, 1	2, 2	1, 2	1, 3	2, 3
2	1, 1	2, 1	2, 3	1, 3	1, 2	2, 2
3	1, 2	2, 2	2, 3	1, 3	1, 1	2, 1
4	1, 2	2, 2	2, 1	1, 1	1, 3	2, 3
5	1, 3	2, 3	2, 1	1, 1	1, 2	2, 2
6	1, 3	2, 3	2, 2	1, 2	1, 1	2, 1
7	2, 1	1, 1	1, 2	2, 2	2, 3	1, 3
8	2, 1	1, 1	1, 3	2, 3	2, 2	1, 2
9	2, 2	1, 2	1, 3	2, 3	2, 1	1, 1
10	2, 2	1, 2	1, 1	2, 1	2, 3	1, 3
11	2, 3	1, 3	1, 1	2, 1	2, 2	1, 2
12	2, 3	1, 3	1, 2	2, 2	2, 1	1, 1

B. Time Estimate

1. Based on 10 consecutive runs per surface (two prewetting passes and eight skids).

		For Two Surfaces
At 20 mph	8 minutes per surface	16 minutes
At 40 mph	12 minutes per surface	24 minutes
At 60 mph	20 minutes per surface	40 minutes

2. Time estimate for one series:

a.	20 mph sequence, tire T_1	16 minutes
	tire change	10 minutes
b.	20 mph sequence, tire T_2	16 minutes
	40 mph sequence, tire T_2	24 minutes
	tire change	10 minutes

c.	40 mph sequence, tire T_1	24 minutes
	60 mph sequence, tire T_1	40 minutes
	tire change	10 minutes
d.	60 mph sequence, tire T_2	40 minutes
		190 minutes

C. Test Day Program

1. With 190 minutes per series, two series can be run per test day, for a total of 240 runs.

2. Each tire type will make 120 runs per day, consisting of 24 wetting passes and 96 skids. Tires are expected to last for 96 skids without excessive wear. Tires will be replaced after 96 skids, i.e., start each test day with a new set of tires. Tires which during the day are found to be shredded or blistered or have differences in groove depth greater than 0.1 inch (see Technical Guidelines) shall be replaced.

D. Number of Test Days

1. Test conditions: Two periods of day, two waterfilm thicknesses, and two states of wear gives $2^3 = 8$ test conditions.

2. Number of test days is equal to 8m, where m is the number each test is repeated. Using m = 4, gives 32 test days.

E. Test Tire Requirements

Based on one tire of each type per test day, 32 tires of each type are needed. However, used tires will be reground to serve as worn tires, so that only half the number of tires is required. To provide for the laboratory tests and spares, 25 tires of each type shall be purchased. Left over tires shall be kept separately for later correlation with other production batches.

F. Complete Daily Design Plan

Table A-2 gives the complete test plan for fixed design variates. The program calls for 32 test days or 64 test series. The first two data sheets of Attachment 1 will be numbered by test series from 1 to 64.

TABLE A-2 DAILY DESIGN PLAN FOR FIXED VARIATES

TEST DAY	TIRE STATE OF WEAR	WATERFILM DEPTH	PAVEMENT SURFACE GROUP	PERIOD OF DAY: I_1 SURFACE WITHIN GROUP		I_1 TIRE AND SPEED SEQUENCE	I_2 SURFACE WITHIN GROUP		I_2 TIRE AND SPEED SEQUENCE
	C	H		No.			No.		
1	1	1	1	2	11	3	11	2	10
2	1	1	2	1	6	11	6	1	66
3	1	1	1	11	2	1	2	11	8
4	1	1	2	6	1	8	1	6	1
5	1	1	1	2	11	6	11	2	11
6	1	1	2	1	6	10	6	1	3
7	1	1	1	11	2	2	2	11	7
8	1	1	2	6	1	7	1	6	2
9	1	2	1	2	11	2	11	2	9
10	1	2	2	1	6	10	6	1	5
11	1	2	1	11	2	6	2	11	7
12	1	2	2	6	1	9	1	6	2
13	1	2	1	2	11	5	11	2	10
14	1	2	2	1	6	2	6	1	6
15	1	2	1	11	2	3	2	11	10
16	1	2	2	6	1	10	1	6	3
17	2	1	1	2	11	6	11	2	7
18	2	1	2	1	6	8	6	1	3
19	2	1	2	11	2	4	2	11	11
20	2	1	2	6	1	7	1	6	6
21	2	1	2	2	11	3	11	2	8
22	2	1	2	1	6	11	6	1	4
23	2	1	1	11	2	5	2	11	12
24	2	1	2	6	1	12	1	6	5
25	2	2	1	2	11	3	11	2	10
26	2	2	2	1	6	11	6	1	6
27	2	2	1	11	2	1	2	11	8
28	2	2	2	6	1	10	1	6	3
29	2	2	1	2	11	6	11	2	11
30	2	2	2	1	6	8	6	1	1
31	2	2	1	11	2	2	2	11	9
32	2	2	2	6	1	10	1	6	5

LABORATORY PROCEDURE

On all runs slip ratio shall be varied from 0.0 to -1.0 to completely define the slip ratio curve.

A. Effect of Load

Use one tire of each type, 24 psi, 0.02 inch water depth. Use 68, 79, 89, and 100 percent of T&RA design load.

B. Effect of Inflation Pressure

Use E 501 tire, 0.02 inch water depth and 1085 lb. laod. Use 20, 24, 28, and 32 psi inflation pressures.

C. Effect of Water Depth

Use one tire of each type, 24 psi and 1085 lb. load. Vary water depth between 0.0 and 0.06 inch inclusive in steps of 0.01 inch.

D. Effect of Groove Depth

Use one tire of each type, 24 psi and 1085 lb. load. Test at same water depth as in (C), at the following groove depths (inches):

	Percent of			Percent of	
	Total	Usable		Total	Usable
0.30 to 0.36	86-103	75-105	0.33 to 0.39	92-109	86-117
0.20 to 0.26	57-75	25-55	0.20 to 0.26	56-75	18-49
0.09 to 0.15	26-43	-30-0.0	0.10 to 0.16	28-45	-34-(-3)

E. Effect of Velocity

Use one tire of each type in the most worn state (D). Run at 24 psi and 2085 lb. load, at water depth of 0.02 and 0.06 inch, at 20, 40, and 60 mph.

APPENDIX B

TEST SURFACES

A representative range of skid resistance and surface texture was desired. This had to be accomplished by four surfaces, according to the test program. Highly abrasive surfaces were to be excluded in order to reduce tire wear and its effect on repeatability. A number of surfaces were available at the Texas Transportation Institute, but with the above limitation only three of those surfaces were found suitable and a fourth surface (No. 11) was specially prepared for these tests.

The four pavements are briefly described in Table B-1, with photographs in Figure B-1. Skid resistance records for the four pavements, at 40 mph, are shown in Figure B-2. The data from 1 to 40 cover the primary testing period (September to December 1974). The last four entries (41 to 44) are tests conducted in May 1975.

Table B-1 Description of test Surfaces

Test Pad Number	Surface Types	Aggregate Weight Percent	Aggregate Type	Aggregate Maximum Size, in.	Texas Highway Department Specifications	Construction Date	Preparation Prior to Testing (Sept. 1970)	Average Texture Depth,** In.
1	Rounded Siliceous Gravel Portland Cement Concrete (Belt Finish)	67	Rounded Siliceous Gravel	1-1/2	(Existing Runway Surfaces)	1953	Cleaned with Water and power broom	0.037
			Siliceous Sand					
2	Clay Filled Tar Emulsion (Jennite) Flushed Seal	33	No Aggregate		Type E*	1968	Scrubbed with water and rubber float	0.012
6	Rounded Siliceous Gravel Surface Treatment (Chip Seal)	100	Rounded Siliceous Gravel	1/2	Grade 4	1970	None	0.050
11	Jennite Flush Seal w. Sand					1974		0.023

*A 3/16 maximum size Type E mix composed of slag and limestone screenings was used as a base for the seal.

**Obtained by putty impression method.

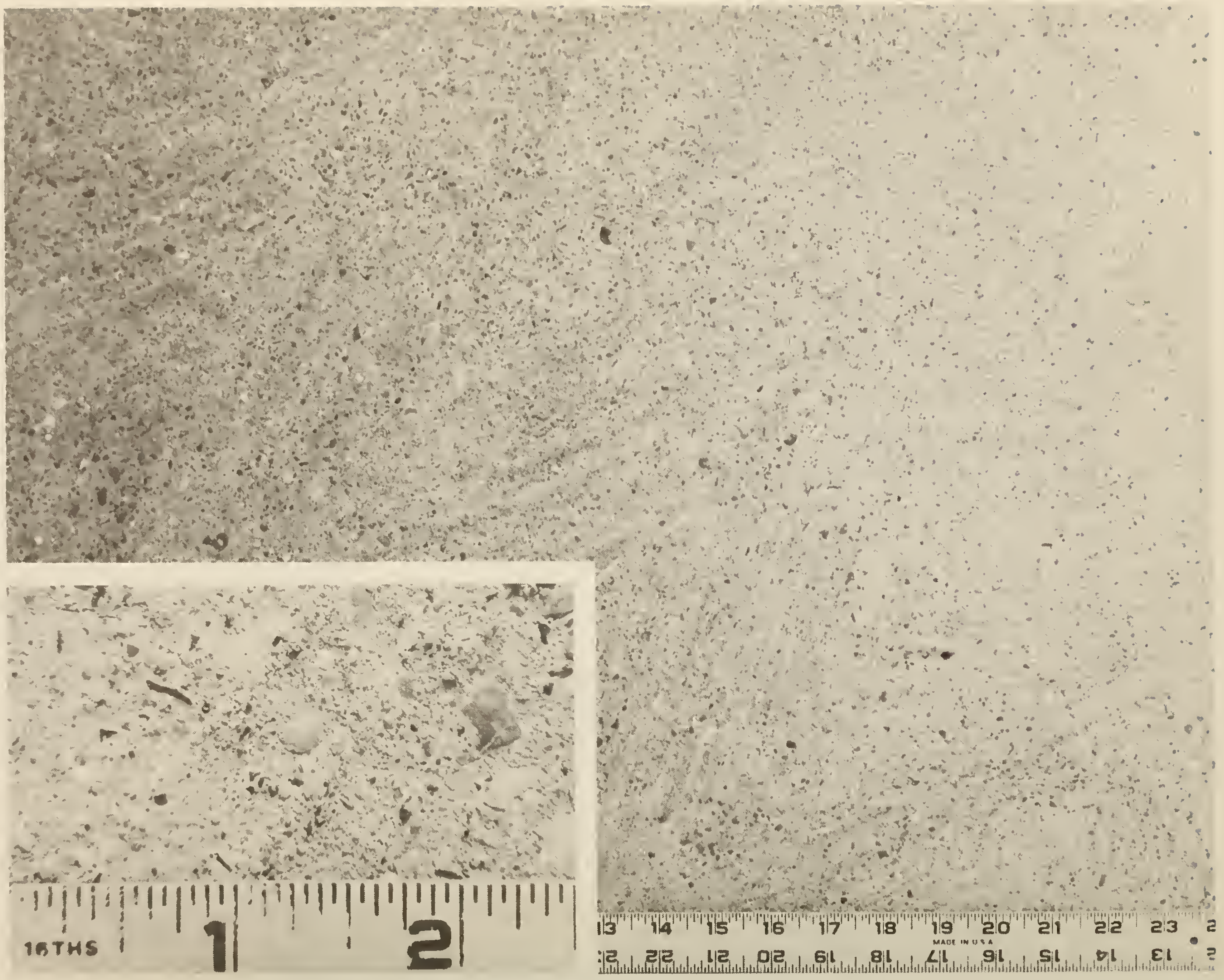

Figure B-1. Surface No. 1, PCC, rounded silicious gravel.

Figure B-1. (continued), Surface No. 2, Jennite

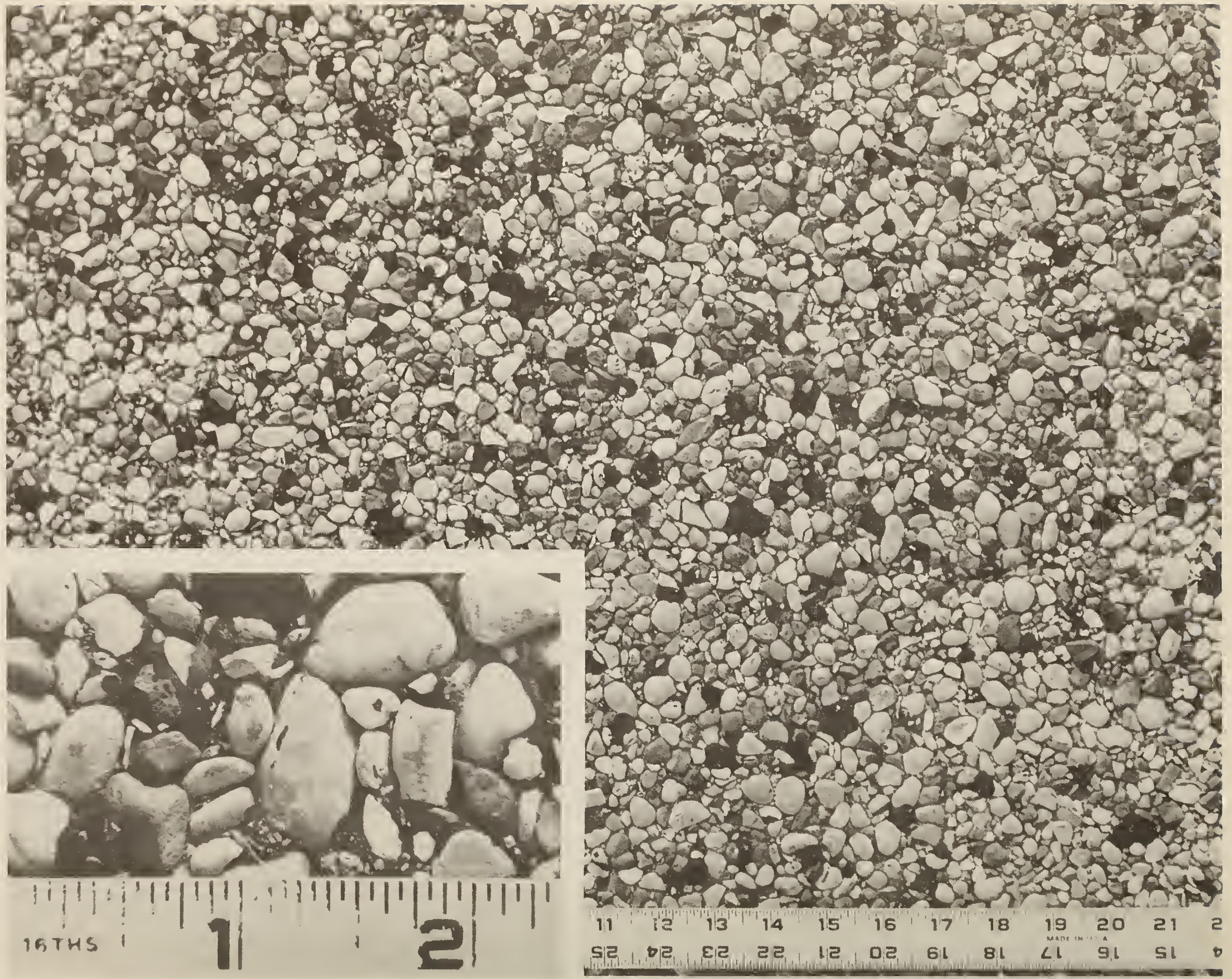

Figure B-1 (continued), Surface No. 6, chip seal, rounded silicious gravel.

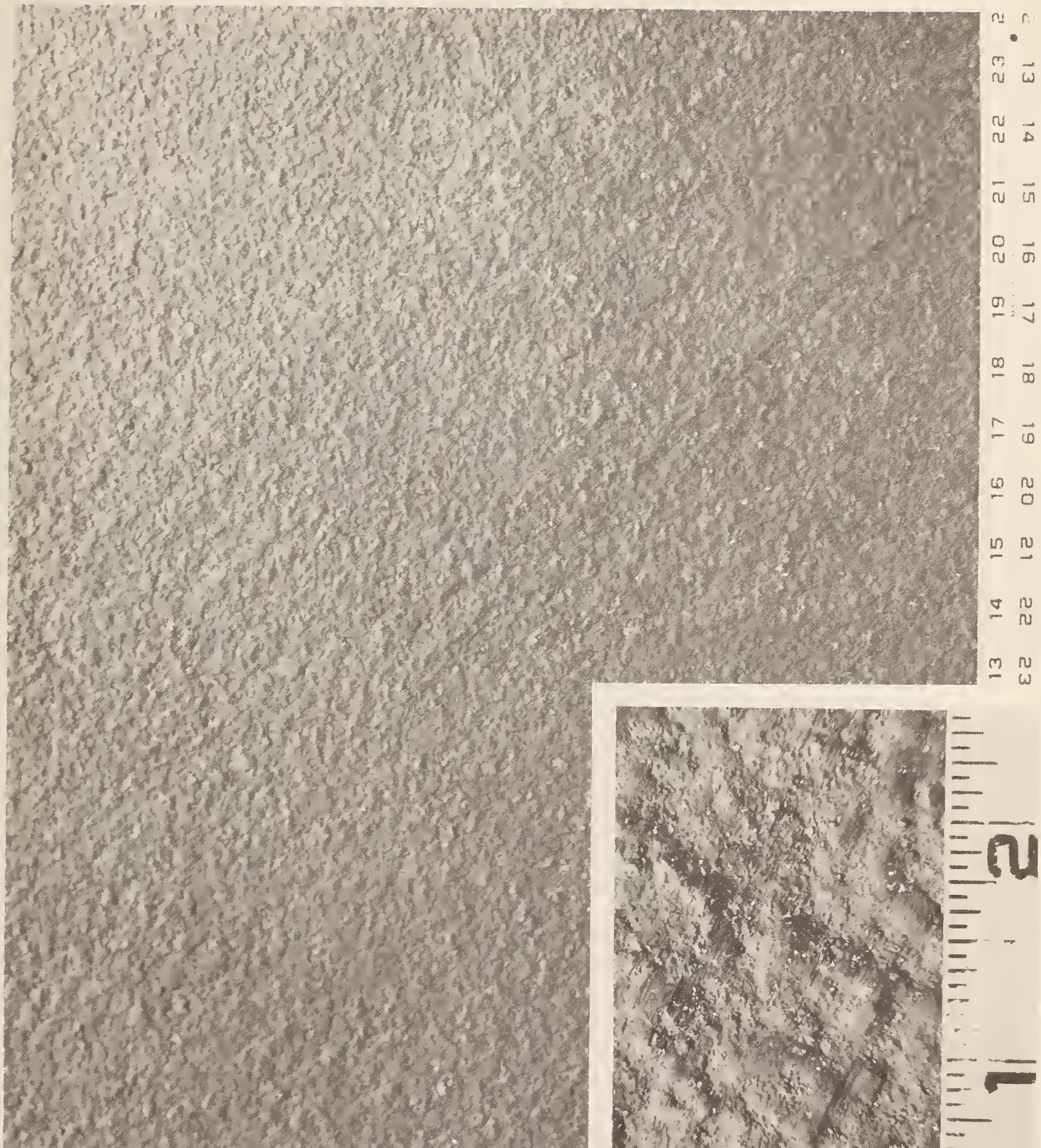

Figure B-1 (continued), Surface No. 11, Jennite flush seal with sand.

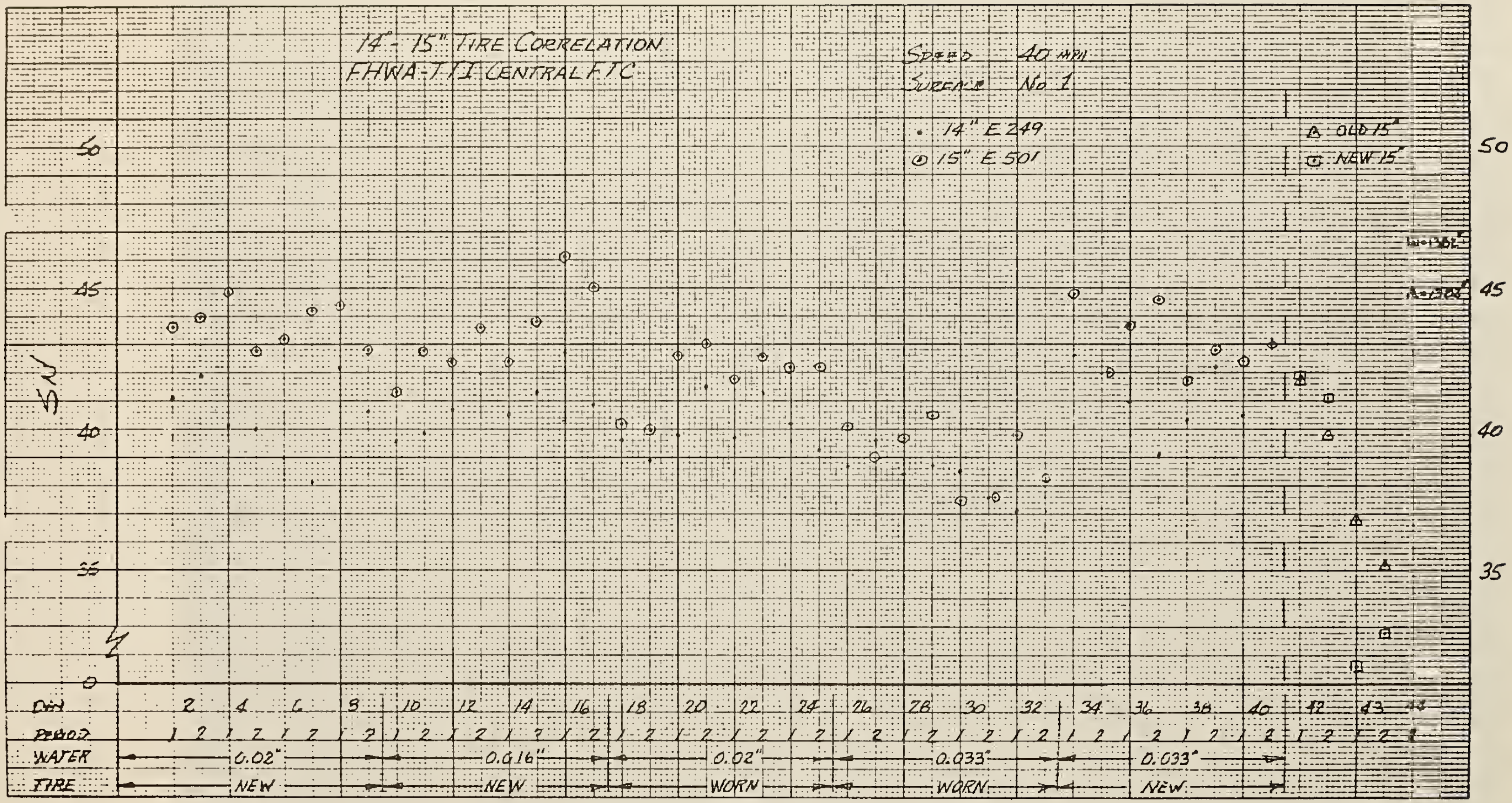

Figure B-2. Skid resistance record, Surface No. 1.

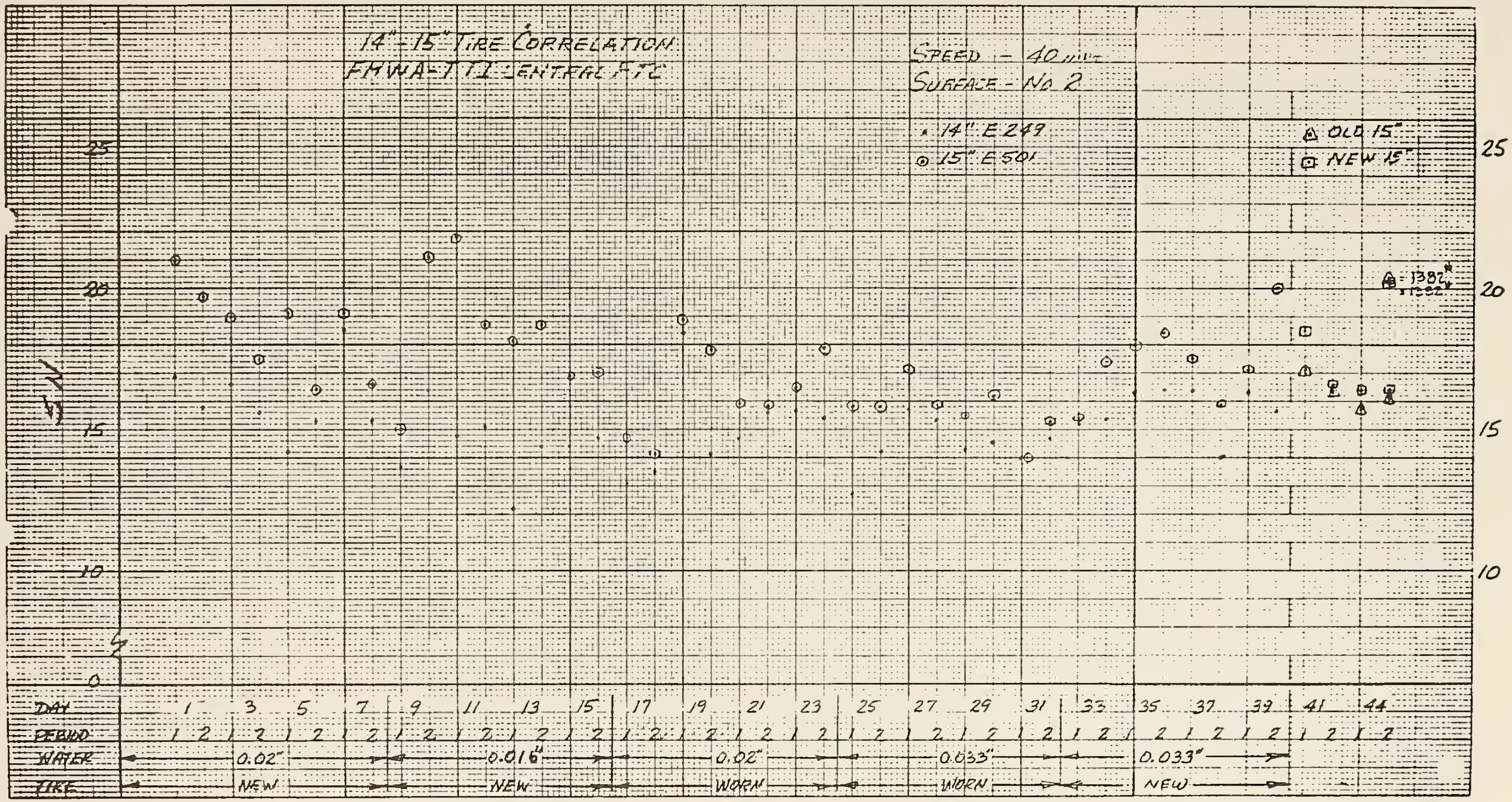

Figure B-2 (continued), Surface No. 2.

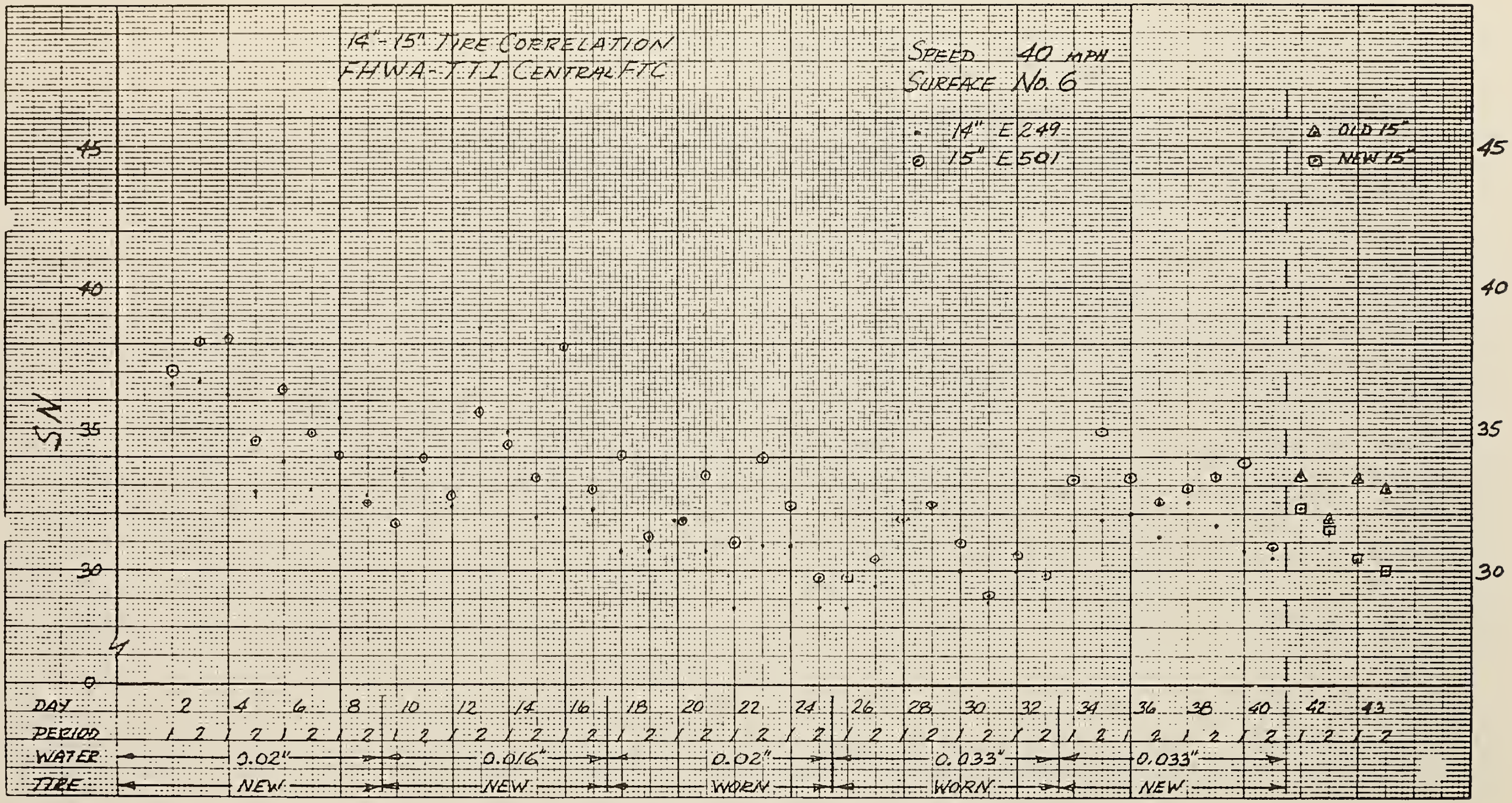

Figure B-2 (continued), Surface No. 6.

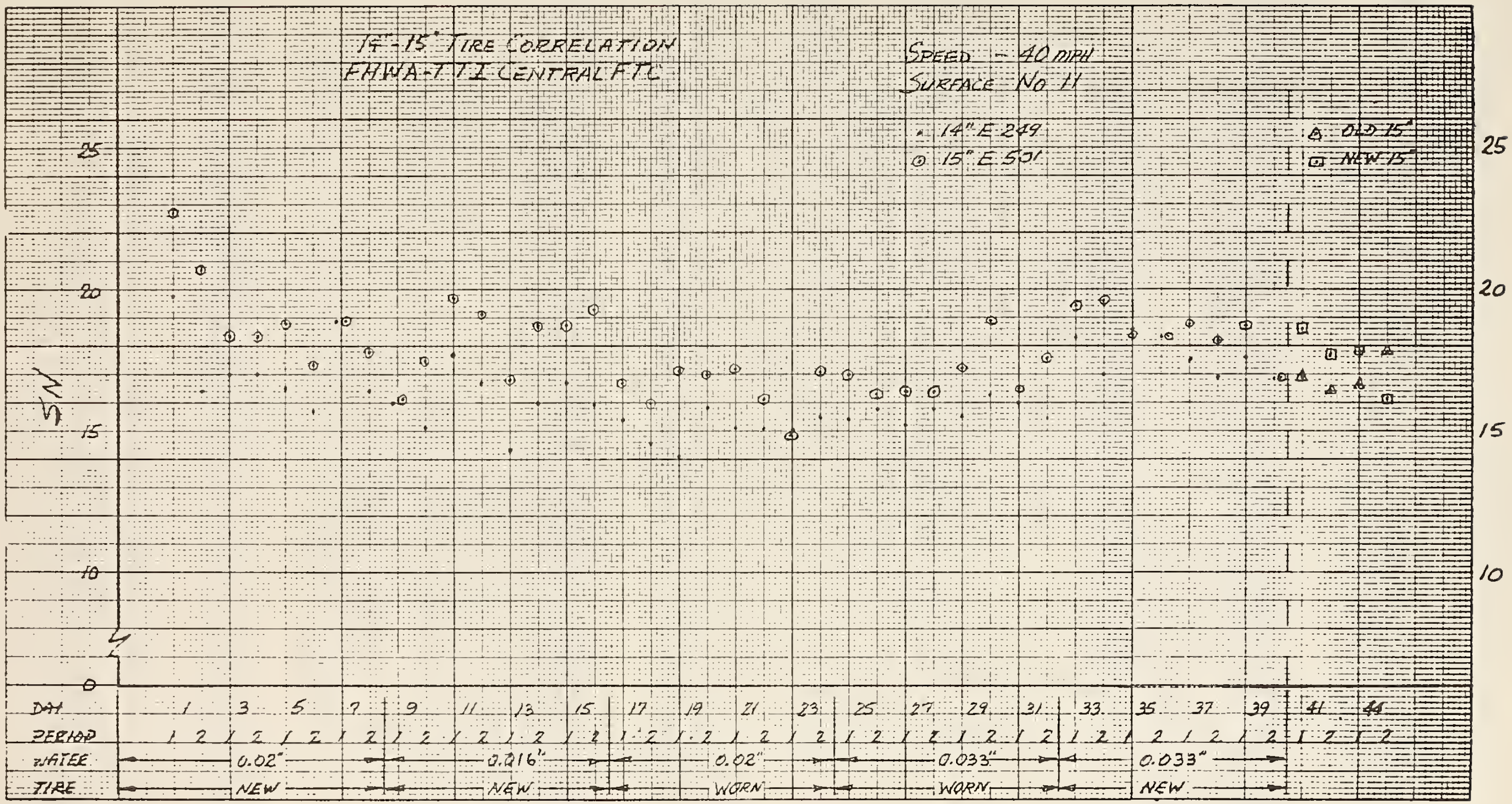

Figure B-2 (continued), Surface No. 11.

APPENDIX C

PLAN FOR STATISTICAL ANALYSIS OF TEST TIRE CORRELATION DATA

C-1. Terminology and Definitions

Factor: Qualitative experimental variable, as opposed to one whose quantitative measurement is directly taken into account in the analysis. A list of factors considered in the experimental plan for skid testing are:

- C Condition of tire (new, worn)
- H Water depth (0.02, 033 inches)
- P Pavement surface type (actually four such types in each of which a given path was selected throughout the experiment)
- V Nominal test speed (20, 40, 60 mph)
- I Interval of day (early morning, early afternoon)
- T Tire type (E 249, E 501)

Dependent Variable: The measured characteristics of major interest in the analysis--in this case, skid number or SN.

Independent Variable: Quantitative experimental variable being examined as to its effect on the value of the dependent variable. The independent variables considered here are:

- X_1 Groove depth of tire
- X_2 Wet pavement temperature
- X_3 Order of skid run (1, 2, ..., 8) within a group

Run: Single performance by the skid vehicle over the test surface.

Group of Runs: Eight repeated runs for any set of factors.

Replication: Four repetitions of the experiment over the full set of factor combinations.

Covariate: An observation on the independent variable for use in the analysis of covariance or as a predictor in the regression equation.

ANOVA: Analysis of variance performed on the measured skid number for the set of factor combinations, ignoring covariates.

CO-ANOVA: "Analysis of covariance " performed on the skid measurements which is an analysis of variance, adjusted by covariates.

Within mean analysis: For each group of eight runs the linear effect of X_3, order of run, is analyzed as to its significance on the skid number.

Tire mean: The average skid number for any group of eight skid runs.

Between mean analysis: An analysis performed on a set of tire means. This could be an ANOVA, CO-ANOVA, calibration (tire correlation) or any other analysis involving SN as the dependent variable.

Calibration: The regression equation that relates two measurement techniques.

Tire calibration: The equation that related the SN for the E 249 tire (SNY) to the SN for the E 501 tire (SNX).

Intercept model: The tire calibration equation that involves the use of a constant (intercept) term in addition to terms involving independent variables.

Non-intercept model: The tire calibration equation which exclusively employs terms that involve independent variables (e.g., SNX, groove depth, temperature).

C-2. General Plan for Statistical Analysis

The statistical analysis can be structured in three parts:

(a) Within tire-mean, at the 96 possible test combinations of factor levels:

- condition of tire (C = 1,2)

- water depth (H = 1,2)

- pavement surface (P = 1,2,3,4)

- interval of day (I = 1,2)

- velocity (V = 1,2,3)

This will be done for each tire type (T = 1,2). Thus, over the 32 days of experiments, there are 384 groups of eight runs for each type or four replications on each of the other 96 treatment (factor) combinations. This will involve order of run, groove depth, and pavement temperature as covariates.

(b) Covariance analysis - on tire type means at the above combinations and same covariates. Also, this will be done with "tire type" as a treatment (factor).

(c) Calibration - calibration or regression lines will be constructed between the two tire types at various conditions.

It is noted that a factor representing day-to-day changes is ignored as well as a factor representing the driver effect. It is presumed that these will be either minimal or indirectly reflected by the various test combinations of factors and covariates already included in the experimental design. It is also noted that the same pavement lane will be used on repeat runs in order to remove within pavement variability. The effect of pavement prewetting will be analyzed by using "order-of-run" as a covariate for the within tire mean analysis. However, this prewetting would render the tests not representative of the usual sequence of tests, leading possibly to lower SN values.

C-3. Within Mean Analysis (within tire type means)

In the Daily Design Plan (see Appendix A) we see for example that under the first treatment combination (C,H,P,I,V) = (1,1,1,1,1) the measurements for tire type T1 as well as for type T2 are repeated on days 1,3,5, and 7. These days serve also to replicate other combinations involving P = 2, I = 2, and V = 2,3 as well. Thus, as we proceed for all 32 days we observe that each treatment combination is replicated four times. Each of the four replications is associated with a different set of covariates, however.

For each tire type, use the following model for an individual run SN value (y):

$$y = \mu_{i_1 \cdots i_5} + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + e$$

where

$\mu_{i_1 i_2 \cdots i_5}$ = ANOVA classification combination of condition, water depth, etc.

(x_1, x_2, x_3) = vector of covariates (groove depth, pavement temperature and order of run)

e = experimental error

It is noted that for any group of eight runs, all quantities are considered constant $(\mu_{i_1 \cdots i_5}, x_1, x_2)$ except x_3, the run order. The model may be written for the first treatment combination (1,1,1,1,1) involving four replications and run-order as follows:

$$y_{11j} = m_{11} + \beta_{(3)} x^{(3)}_{11j} + e_{11j}$$

$$y_{12j} = m_{12} + \beta_{(3)} x^{(3)}_{12j} + e_{12j}$$

$$y_{13j} = m_{13} + \beta_{(3)} x^{(3)}_{13j} + e_{13j}$$

$$y_{14j} = m_{14} + \beta_{(4)} x^{(3)}_{14j} + e_{14j}$$

where

$$m_{11} = \mu_{i_1 \cdots i_5} + \beta_1 x^{(1)}_{11} + \beta_2 x^{(2)}_{12}$$

is a constant for the first group.

$$m_{12} = \mu_{i_1 \cdots i_5} + \beta_1 x^{(1)}_{12} + \beta_2 x^{(2)}_{12}$$

is a constant for the second group, etc.

while

$$x_3 = j = 1,2,\ldots,8.$$

Proceeding to the next treatment combination we see a new set of observations consisting of four groups (of eight runs) that can be modelled as the second treatment combination in four replications:

$$y_{21j} = m_{21} + \beta_{(3)} x^{(3)}_{21j} + e_{21j}$$

$$\cdots = \cdots$$

$$\cdots = \cdots$$

$$y_{24j} = m_{24} + \beta_{(3)} x^{(3)}_{24j} + e_{24j}$$

This can be continued until the last of the 96 replications of treatment combinations are obtained, viz:

$$y_{96,1,j} = m_{96,1} + \beta_{(3)} x^{(3)}_{96,1} + e_{96,1,j}$$

$$\cdots = \cdots$$

$$\cdots = \cdots$$

$$y_{96,4,j} = m_{96,4} + \beta_{(3)} x^{(3)}_{96,4} + e_{96,4,j}$$

There are three kinds of tests or questions one would like to make on $\beta_{(3)}$

(1) For any group, does $\beta_{(3)}$ exist?

(2) Do the $\beta_{(3)}$'s differ from group to group within any treatment combination (or set of replications)?

(3) If the $\beta_{(3)}$'s are the same for any treatment combination, do they differ from combination to combination?

(4) If differences exist, what adjustments have to be made in order to perform an analysis on group means?

<u>Procedure for Testing (1)</u>

One wishes to test the general model of differing run-order effects

$$y_{ikj} = m_{ik} + \beta_{ik} x_{ikj} + e_{ikj} \tag{C-1}$$

where

$$x_{ikj} = j \quad (j = 1, 2, \ldots, 8)$$

against the null hypothesis model

$$y_{ikj} = m_{ik} + e_{ikj} \tag{C-2}$$

for

$$i = 1, 2, \ldots, 96; \quad k = 1,2,3,4; \quad j = 1, 2, \ldots, 8.$$

Equation (C-1) corresponds to each group of eight runs having its own regression with respect to order of run (x_3), while Eq. (C-2) corresponds to all of the β's being simultaneously equal to zero and m_{ik} is the same for each group.

Standard regression analysis procedures provide the following formulas:

R_1 = Reduction in SS (due to individual b's adjusted for means)

$$= \sum_{i=1}^{96} \sum_{k=1}^{4} \left\{ \sum_{j=1}^{8} (x_j - \bar{x})\, y_{ikj} \right\}^2 \Big/ \sum_{j=1}^{8} (x_j - \bar{x})^2$$

$$= \frac{1}{42} \sum_i \sum_k \left\{ \sum_j (x_j - \bar{x})\, y_{ikj} \right\}^2$$

Since

$$\bar{x} = \sum_j j/8 = 9/2$$

while

$$\{x_j - \bar{x}\} = \{-3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5\}.$$

Thus

$$R_1 = \sum_i^{96} \sum_k^{4} \hat{\beta}_{ik} \text{ num } \hat{\beta}_{ik} \qquad \text{(C-3)}$$

where

$$\hat{\beta}_{ik} = \sum_{j=1}^{8} (x_j - \bar{x})\, y_{ikj} \Big/ \sum_{j=1}^{8} (x_j - \bar{x})^2$$

and

$$\text{num } \hat{\beta}_{ik} = \sum_{j=1}^{8} (x_j - \bar{x})\, y_{ikj}$$

E_1 = Error S.S.

$$= \sum_{i=1}^{96} \sum_{k=1}^{4} \left\{ \sum_{1}^{8} (y_{ikj} - \bar{y}_{ik})^2 - \hat{\beta}_{ik} \text{ num } \hat{\beta}_{ik} \right\}. \qquad \text{(C-4)}$$

R_1 has 384 d.f. while E has (384) x 6 or 2304 d.f. The test statistic for the null hypothesis (C-2) is

$$F_{384,\,2304} = \frac{R_1/384}{E_1/2304}$$

If F is significant we reject H_0.

Calculate F for each of the two tire types.

<u>Procedure for Testing (2)</u>

If Test (1) above is rejected, then another item of interest is: Are the order-of-run regressions equal within individual replications?

We thus desire to test the hypothesis of replication commonality:

H_0 : $\beta_{ik} = \beta_i$ $(k = 1, 2, 3, 4)$ (REPLICATION COMMONALITY)

for each i and k

(i.e. $\beta_{11} = \beta_{12} = \beta_{13} = \beta_{14};$ $\beta_{21} = \beta_{22} = \beta_{23} = \beta_{24}; \ldots; \ldots;$

until finally $\beta_{96,1} = \beta_{96,2} = \beta_{96,3} = \beta_{96,4}$).

First calculate the reduction in regression SS under replication commonality for each combination (96 in all). This reduction is given by

$$R_2 = \sum_{1}^{96} \hat{\beta}_i \text{ num } \hat{\beta}_i$$

where

$$\hat{\beta}_i = \frac{\sum_{k=1}^{4} \sum_{j=1}^{8} (x_{ikj} - \bar{x}_{ik})\, y_{ikj}}{\sum_{k=1}^{4} \sum_{j=1}^{8} (x_{ikj} - \bar{x}_{ik})^2} = \frac{\text{num } \hat{\beta}_i}{\text{den } \hat{\beta}_i}$$

and

$$\left(\begin{aligned} \text{num } \hat{\beta}_i &= \sum_{k=1}^{4} \text{num } \hat{\beta}_{ik} \\ \text{den } \hat{\beta}_i &= \sum_{k=1}^{4} \text{den } \hat{\beta}_{ik} = 4 \times 42 = 168\,. \end{aligned} \right)$$

Note that for each (i, k) combination

$$\sum_{j=1}^{8} (x_{ikj} - x_{kj})^2 = 42$$

Also note that R_2 contains 96 df.

Then calculate the "added" reduction R of SS of individual regressions from the SS due to assuming a common regression. This is given by

$$R = R_1 - R_2 \text{ (representing } 384 - 96 = 288 \text{ d.f.)}$$

Then

$$F_{288,\,2304} = \frac{(R_1 - R_2)/288}{E_1/2304}$$

is the appropriate statistic to test

$$H_0 \quad \beta_{ik} = \beta_{ik'} = \beta_i \quad (i = 1, 2, \ldots, 96)$$

$$\text{for any } k \neq k' = 1, 2, 3, 4 .$$

Based on the results for each tire type, one can form consolidated tests of the respective hypotheses on both types. Inasmuch as printouts will be obtained for R , R_1, R_2, and E on both types, such tests will be easy to perform. On the basis of such tests we will know how to structure "order of run" into the covariance model for means.

Procedure for Testing (3)

If Test (2) accepts the null hypothesis that we do have commonality within replications for the "order of run" regression coefficient, we might try to test the universality of the commonality over all treatment combinations, i.e.,

$$H_0 : \beta_{ik} = \beta \quad \text{for all } i, k$$

versus the alternative

$$\beta_{ik} \neq \beta_{i', k'} \quad \text{for all } i \neq i', k \neq k'$$

First find the regression sum of squares due to common β . This is given by

$$R_3 = \hat{\beta} \text{ num } \hat{\beta}$$

corresponding to one d.f., where

$$\hat{\beta} = \sum_{1}^{96} \text{num } \beta_i \Big/ \sum_{1}^{96} \text{den } \hat{\beta}_i$$

The appropriate F test for the adoption of one regression coefficient is:

$$F_{383,\,2304} = \frac{(R_1 - R_3)/383}{E_1/2304}$$

If $F_{383,2304} > F_{383,2304}(.05)$,
then the above commonality hypothesis is rejected.

Effect of Order of Run: Procedure for Within Mean Adjustments

We will know how to structure this effect in the mean of a group of eight runs under possible outcomes of previous tests for cases (a) through (d) below:

(a) If $\beta^{(3)}_{ik} = 0$ for all i, k $(i = 1,2,\ldots,96;\ k = 1,2,3,4)$ then it is obvious that an analysis of covariance on means can be immediately performed, since the model is the same as (C-2) or

$$y_{ikj} = \mu_{i_1 \ldots i_5} + \beta_1 x^{(1)}_{ik} + \beta_2 x^{(2)}_{ik} + e_{ikj} \qquad \text{(C-5)}$$

where $i_1, \ldots, i_5$ depend on i, for each $j = 1,2,\ldots\ldots,8$
We can model the group mean by

$$y_{ik} = \mu_{i_1 \cdots i_5} + \beta_1 x^{(1)}_{ik} + \beta_1 x^{(2)}_{ik} + e_{ik} \qquad \text{(C-6)}$$

where

$$y_{ik} = 1/8 \sum_{j=1}^{8} y_{ikj} \quad \text{and} \quad e_{ik} = 1/8 \sum_{j=1}^{8} e_{ikj}$$

The consolidated CO-ANOVA table for the 96 treatment combinations, three covariates and involving four replications can be represented by a within-mean and between-mean analysis as follows:

Source	d.f.
Between Means	(383)
Covariates (groove depth (β_1), temperature (β_2))	2
Treatments (main effects and interactions adjusted by β_1 and β_2)	95
Between Mean Error (adjusted by β_1 and β_2)	286
Within Means	(2688)
Order-of-Run (Covariate) β_3	384
Within Mean Error	2304

(b) If $\beta_{ik}^{(3)} = \beta \neq 0$ for all i,k, then again a covariance analysis on means can be performed on the model

$$y_{ik} = \mu_{i_1 \ldots i_5} + \beta_1 x_{ik}^{(1)} + \beta_2 x_{ik}^{(2)} + 4.5\beta \qquad \text{(C-7)}$$

However

$$\mu_{i_1 \cdots i_5} = \gamma + \alpha_{i_1} + \alpha_{i_2} + \ldots$$

Hence, the constant 4.5β automatically appears each time γ appears so that the general mean effect is actually " $\gamma + 4.5\beta$." Hence, all we need to do is to perform the same calculations as in 1(a) above, with the same CO-ANØVA and with the understanding that the general mean includes the run order effect.

Note: Since this is done for each tire type, then if $\beta^{(3)}$ is the same for both types, the run order effect is nullified in such comparison. Also, if one wishes to take cut the β effect, then the CC-ANOVA could be run on the adjusted means: $y_{ik} - 4.5\hat{\beta} = y_{ik}^*$ where $\hat{\beta}$ was estimated in the order-of-run regression analysis.

(c) If $\beta_{ik}^{(3)} = \beta_i^{(3)}$ for k = 1,2,3,4 , then for each treatment combination i, we have a new run order effect. For the mean y_{ik} , the model is

$$y_{ik} = \mu_{i_1 \cdots i_5} + \beta_1 x_{ik}^{(1)} + \beta_2 x_{ik}^{(2)} + 4.5\,\beta_i \qquad \text{(C-8)}$$

where $\mu_{i_1 \cdots i_5} = \mu_i$

so that is confounded in some way with μ_i . That is, one would not be able to distinghish the contribution of β_i and μ_i in the model.

Cne way to counteract this is to adjust each y by $4.5\,\hat{\beta}_i$. That is, run a CO-ANOVA on the adjusted mean

$$y_{ik}^* = y_{ik} - 4.5\,\hat{\beta}_i^{(3)}$$

in order to find the adjusted treatement and regression effects. The same format for the CO-ANOVA could be run cn the y_{ik}^*'s as in case (b).

(d) If $\beta_{ik}^{(3)}$ depends on i and k then we are confronted with 96 x 4 = 384 parameters to contend with. An analysis on means can be performed if we adjust them for each i and k,

i.e., $$y_{ik}^{*} = y_{ik} - 4.5\ \hat{\beta}_{ij}^{(3)} \tag{C-9}$$

and the same CO-ANOVA can be performed as in cases (b) and (c).

C-4. Between Mean Analysis (CO-ANOVA & ANOVA)

The CO-ANOVA will be performed on the treatment combination means, appropriately adjusted, if necessary, as explained in the previous section. Three CO-ANOVA's will be developed: Tire Type 1, Tire Type 2, and a full CO-ANOVA involving "tire types" as a treatment factor in order to study its possible interactions with other treatment factors such as pavement surface or velocity.

For either Type CO-ANOVA, the analysis will involve an orthogonal 2^3x 3 x 4 factorial design in four replications. The factors in the design correspond to Condition of Tire (C), Water Depth (H), Interval of Day (I), Velocity (V), and Pavement Surface (P). Since the analysis in on the mean (adusted for β^3 , if necessary) there will be only two covariates: Groove Depth (x_1) and Pavement Temperature (x_2).
For the full CO-ANOVA employing Types as factor, the analysis would involve a full (orthogonal) 2^4 x 3 x 4 factorial design in four replicates with the other treatments and covariates being the same as before.

Note: Depending on the utility of outputs from available CO-ANOVA programs, it may be useful to utilize an ANOVA program that ignores the two covariates. Also it is considered that the two factors, C (Condition of Tire) and I (Interval of Day) could be"eliminated"in view of their strong respective correlations to the two covariates: groove depth and temperature. This would increase the available number of error d.f.

Either tire type CO-ANOVA on the means can be represented as follows: (2 x 3 x 4 factorial with 16 replications)

Source	d.f.
Main effects	(6)
H	1
V	2
P	3
Two-Way Interactions	(11)
H x V	2
H x P	3
V x P	6
Three-Way Interactions	(6)
H x V x P	6
Between Mean Error (E)	358

It is noted that the number of degrees of freedom in E are computed by subtracting the number (two in all) of regression coefficients used in the between mean analysis from the number of degrees of freedom in an ordinary ANOVA (where d.f. = 16 x 24 - 24 = 360).

Higher order interactions will be examined individually, although they are aggregated in the above CO-ANOVA table. We are to be reminded that all effects as well as between mean error sums of squares have been adjusted by the two covariates: mean groove depth and pavement temperature. It is also considered that interpretation of significant higher order interactions is usually quite difficult and therefore any discussion on this aspect will be postponed until such time as the data is presented.

For the present let us discuss the interesting features or consequences of a significant main effect and two-factor interaction within a tire type. It can be seen that the appearance of a significant main effect would not be troublesome at all. All that it would indicate, for example, is that a SN reading at one velocity would generally be different than at another velocity, if V was significant. One would expect that certain main effects as P and V would be significant, even (or especially) after adjustment by x_1 and x_2 . In addition it will be interesting to examine the effect of water depth H.

The appearance of a significant two-factor interaction such as P x V would indicate that for the particular tire type, differences among pavements are not the same from speed to speed and vice versa. This would not affect tire calibration as long as the P x V interaction would behave the same way for the other tire type. Hence one would be more concerned, for calibration purposes, with interactions involving T as a factor - as will be attempted in a full CO-ANOVA involving T as a factor.

Also if it happens, for example, that a main effect such as H is not significant, while its interaction with velocity H x V, is significant then one would have to conduct a partitioned ANOVA to examine H further in order to reach a more valid conclusion as to non-significance. That is, perform two CO-ANOVA's if H x C is significant each one being under separate conditions C = 1, C = 2.

Another item of interest is the error mean square for each tire type. This would be produced from the CO-ANOVA's for each T. This would enable us to compare the within variance for each mean.

From each of the two CO-ANOVA's one can compare (run a test) on the two sets of covariates β_{11}, β_{12} and β_{21}, β_{22} for each tire type. It would perhaps be easiest to make the comparison from the following computations

R_1 = Reduction in Error SS due to β_1, β_2 within Type 1

R_2 = Reduction in Error SS due to β_1, β_2 within Type 2

R_c = Reduction in Error SS due to common (β_1, β_2) for the combined CO-ANOVA)

Then "$R_1 + R_2 - R_c$" is the added Reduction in Error SS due to differences between the two sets with 2 d.f.

The full CO-ANOVA, in conjunction with the two individual CO-ANOVA's, would be used to test commonality in the covariates x_1 and x_2. Also it would be used to detect interaction of Tire Types with other factors. Details of this were previously discussed. It is contemplated that various other aspects that require interpretation will arise when the data set is presented for analysis.

C-5. Calibrations Between Tire Types

The determination of the correlation between two tire types can be formally considered as a "calibration" problem. That is, suppose SNX is an observed reading of the skid number for the E 501 tire. It is then desired to form a "calibrated scale" from this initial reading in order to obtain the measured quantity SNY, which is the skid number for the E 249 tire. The desired scale is usually produced in terms of a straight line fit that is used as a "calibration curve" for the two variables with the basic linear equation

$$SNY = a_0 + a_1 \; SNX \qquad \text{(C-10)}$$

The right hand side of Eq. (C-8) is analogous to a calibrated gauge from which one reads off the value of SNY. However, in the development of such an equation there is encountered relatively large experimental errors for both variables. Thus the resulting fit would not ideally satisfy the fullest requirement for a calibration line that involves gauging without appreciable error. However, if the number of observations used to determine the fit is large and if the range of the readings for SNX is broad enough then one can establish a useful calibration line for predicting SNY from SNX.

Under usual assumptions of straight line fitting it is only the dependent variable that is subject to error. However, it was shown by Berkson (1) that one can fit a straight line to paired sets of data even though the measurements for both variables (SNX and SNY) are subject to experimental error. Hence the variable SNX is a "controlled" quantity, in the sense that for each measurement of SNY_i, the corresponding value of SNX is "set" at an assigned value SNX_i, or as close to SNX_i as is experimentally feasible. Reference 2 (Mandel), states "From a practical point of view, the Berkson model . . . assures us that . . . we may apply the method of least squares for straight line fitting as though the controlled variables were free of error, even though this may not be the case."

Calibrations will be formed between SNY as the dependent variable and SNX, D and T as the independent variables where

D = G 249 - G 501 (the observed difference in groove depths between the E 249 and the E 501 tire means)

and

> $T = T\ 249 - T\ 501$ (the observed difference in wet pavement temperatures between the E 249 and E 501 tire means)

The equation to be fitted is linear of the form

$$SNY = a_0 + a_1\, SNX + a_2\, D + a_3\, T \qquad \text{(C-11)}$$

wherein the possibility of second order and higher terms are desregarded.

Separate categories of equations will be obtained for each test speed condition, each pavement surface as well as a consolidated equation that utilizes all the 384 sets of data means (SNY, SNX, D, T).

For each of the eight categories, an analysis will be conducted as to the significance of the linear components. In addition, an extended analysis will be made to develop the appropriate equation to be used for calibration. Inasmuch as such calibration constitutes a prediction -- selection of an appropriate equation will be based on the variability of a prediction. The statisical literature will be investigated to provide a suitable criterion upon which to base the selection (3), (4), (5).

The criterion will be applied to the intercept term (a_0) itself as well as each of the other terms. Candidate models that involve the intercept are referred to as "intercept models" while those that do not involve a_0 are referred to as "non-intercept" models. In effect dropping a_0 from the equation would force the prediction line to pass through the origin, in agreement with the underlying physical law governing such relations. Of interest to our problem is the following statement by Helms (2):

> "Although this (inclusion of intercept terms) is common practice . . ., our experience has indicated intercept terms are frequently primary contributors to variance but their absence often leads to only small contributions to bias. To systematically ignore the possibility of deleting the intercept terms seems to be unjustified."

A relatively simple criterion upon which to base our selection is to evaluate for each prediction (calibration) equation the following expression (over a set of nominal SNX values where D = T = 0):

$$s_y^2 = s_e^2 + x'Wx \qquad \text{(C-12)}$$

where

s_e^2 = estimated error variance based on the fitted calibration model to the set of means based on eight observations.

x' = $(1, a_1)$ for the intercept model or (a_1') for the non-intercept model.

x = the transpose vector of x'.

and w = the variance-covariance matrix of (a_0, a_1) if the intercept model is applied or $\text{Var}\, a_1'$ if the model is non-intercept.

Based on preliminary calculations, it seems that the non-intercept class of models would be favored using criterion (C-10) since the estimated prediction variances s_v^2, are similar. If one were to strictly apply the A.E.V. criterion (Helms), one would tend to drop the terms D and T from the prediction. However, based on other considerations, it may be preferable to retain both D and T. One form of the A.E.V. criterion is:

$$\text{A.E.V.} = k s_e^2 / N \qquad \text{(C-13)}$$

where

k = number of terms in the "prediction" equation upon which s_e^2 is based,

and

N = the sample size.

The A.E.V. criterion should be applied within each separate category to select the appropriate model. That is, it is not proper procedure to compare two different categories of models such as the "20 mph" and "40 mph" models. The selection of the category would depend on its experimental applicability - on the nominal speed of the skid tester perhaps or the pavement surface, if desired.

Althcugh the A.E.V. criterion is structured differently than that shown in (C-12), it also favors the non-intercept models for the skid calibration data.

The calibrations were made to give a scale for SNY in terms of SNX. It may be desired to utilize the inverted equation to (C-11):

$$SNX = (SNY - a_0 - a_2 D - a_3 T)/a_1 \tag{C-14}$$

where SNY assumes the role of a predictor in place of SNX. In its present form, the expectation of SNX would be difficult to examine directly since the term appearing in the denominator (a_1) is correlated with a_0, a_2 and a_3 that appear in the numerator. If D and T are set equal to zero, the equation simplifies to read

$$SNX = (SNY - a_0)/a_1 \tag{C-15}$$

where we are faced with the same correlation difficulty.

However, if the original prediction is developed in the alternate model:

$$\begin{aligned} SNY &= b_0 + b_1 \, (SNX - S\overline{NX}) \\ &= (b_0 - b_1 \; S\overline{NX}) + b_1 \; SNX \end{aligned} \tag{C-16}$$

where $E(b_0) = \beta_0$ and $E(b_1) = \beta_1$.

it turns out that the new regression fit (C-16) is equivalent to that of (C-10) since:

$$b_1 = a_1, b_0 = a_0 + b_1 \, S\overline{NX}$$

Moreover, regression theory informs us that b_0 and b_1 are statistically independent. Hence we have

$$SNX = S\overline{NX} + (SNY - b_0)/b_1 \tag{C-17}$$

for the inverted equation, where SNY is the predictor and and $S\overline{NX}$ is the mean skid number cf X in the concluded experiment (treated as a controlled variable or constant).

The last equation can be put in the alternate form

$$\begin{aligned} SNX &= (SNY - b_0 + b_1 \; S\overline{NX})/b_1 \\ &= (SNY - a_0)/a_1 \end{aligned} \tag{C-18}$$

where a_0 and a_1 are corrrelated.

Even in the form (C-17) there is the additional problem of the estimator of β_1, appearing in the denominator. Although b_0, b_1 and SNY are mutually independent, the expectation of SNX, E (SNX) does not turn out to be $(SNY - \alpha_0)/\alpha_1$, as desired since

$$E(1/b_1) \neq [E(b_1)]^{-1} \text{ .}$$

However, if the sample variance of SNX is large relative to the conditional error variance of SNY, the bias may be neglected.

<u>Technical Note on Regression</u>:

<u>Relationship Between Unadjusted and Adjusted (for Slope) Sample Variance</u>

The <u>unadjusted sample variance</u> is given by the formula

$$(1) \quad s_1^2 = \sum_1^n (y_1 - \bar{y})^2 / (n-1)$$

while the slope-adjusted sample variance is given by

$$(2) \quad s_2 = \sum_1^n \left\{ y_i - \bar{y} - \hat{b}(x_i - \bar{x}) \right\}^2 / (n-2)$$

where

$$\hat{b} = \sum_1^n (x_i - \bar{x}) y_i \Big/ \sum_1^n (x_i - \bar{x})^2 \text{ .}$$

Formulas (1) and (2) may be written as

$$(1) \quad (n-1)\, s_1^2 = \sum_1^n (y_i - \bar{y})^2$$

$$(2) \quad (n-2)\, s_2^2 = \sum_1^n (y_i - y) - \hat{b} \text{ num } \hat{b}$$

where

$$\text{num } b = \sum_1^n (x_i - \bar{x})\, y_i \text{ .}$$

Hence we have

$$(3) \quad (n-2)\, s_2^2 = (n-1)\, s_1^2 - \hat{b} \text{ num } \hat{b}$$

or

$$(4) \quad (n-1)\, s_1^2 = (n-2)\, s_2^2 + \hat{b} \text{ num } \hat{b}$$

* Equation (4) informs us that the within SS <u>ignoring the slope</u> is always larger than the within SS adjusted by the slope:
(since $\hat{b}$ num $\hat{b} > 0$).

* This is not necessarily true for the <u>respective mean W.S.S. and W.S.S. (adjusted).</u>

Proof From (4):

$$(5) \quad s_1^2 = s_2 - \frac{1}{n-1} s_2^2 + \frac{\hat{b} \text{ num } \hat{b}}{n-1}$$

Hence $s_1^2 \geq s_2^2$ provided that

$$\hat{b} \text{ num } \hat{b} \geq s_2^2 \qquad \text{(I)}$$

Otherwise, if (I) does not hold, then we may observe that the adjusted W.S.S. (s_2^2) is greater than (s_1^2).

This result is most interesting. It tells us that if we insert an "unneeded parameter" in the regression equation, then the mean error may increase (which is a reversal of what we would like). It instructs us to be "parsimonious" in the inclusion of unncessary parameters in our regression model.

The above argument can easily be extended to apply to more than one independent variable.

REFERENCES

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3. R.C. Walls and D.L. Weeks, "A Note on the Variance of a Predicted Response", The American Statistician, 33(3) 24-26 (1969)
4. R.W. Helms, " The Average Estimated Variance Criterion for the Selection-of Variables Problem in General Linear Models", Technometrics 16 (2) 261-273 (1974)
5. D.M. Allen, "The Relationship Between Variable Selection and Data Augmentation and a Method for Prediction", Technometrics 16 (1) 125-128 (1974)

APPENDIX D

DESCRIPTION OF ANALYSIS SOFTWARE

This appendix is a partial documentation of the software used in the three stages of analysis and some observations on the programs used.

The first step in the analysis called for testing of run-order effect between the eight consecutive skid runs. A Fortran program was developed to statistically measure this run-order effect, summarize the data for each test series and output mean data (of eight skid runs) for later analyses. The program input called for punched cards, each containing the eight consecutive skid numbers tire groove depth, pavement temperature, and numeric codes for the test conditions. The output was divided into three parts: statistical summaries of the run-order analysis, table summaries of the data by test condition, and punched output of the mean skid numbers for later analysis. The run-order analysis, Figure D-1, summarizes the test conditions and various computed statistics for each set of eight consecutive skid runs. The nomenclature used to describe these statistics was chosen to conveniently identify the quantities which were being estimated. However, it may not be recognized as standard terminology. Figure D-2, gives the formulas used to estimate these statistics. The table summaries of the data, i.e., means and variances, are included in Appendix E. The final output from this program consisted of punched cards containing the mean skid number (average of eight runs), codes to identify test series, tire type, condition, speed, surface, etc. Thus, each original set of data corresponding to a given water depth and tire condition, consisting of 1536 data points was reduced to 192 (96 points for each tire). These card data were used for subsequent analysis of variance and calibration runs.

Because of its restricted application to this problem only, the order-of-run program is not formally documented here. It could, however, be modified for similar applications. Additional information on the program may be requested from the authors of this report. A detailed discussion of the statistical rationale for the program can be found in Appendix C.

```
REGRESSION ANALYSIS OF TEST TIRE CALIBRATION DATA
        (RUN DATA  VS  RUN ORDER)

    TIRE          :  E501
    TEST SERIES   :  40-2
    TIME          :  PM
    REPLICATION   :  4
    CONDITION     :  NEW
    SPEED         :  40
    SURFACE       :   5
    WATER DEPTH   :  .033
    ROAD TEMP     :   60 F
    GROOVE DEPTH  :  0.364 IN.

STATISTICAL ANALYSIS :
---------------------

    MEAN SKID RESISTANCE =   30.80
    VARIANCE             =    1.43
    STD. DEV.            =    1.19
    NUMBIJ  SS(X,Y)      =  -12.00
    DENBIJ  SS(X,X)      =   42.00
    BIJ                  =   -0.29
    BIJ X NUMBIJ         =    3.43
    ERROR S.S.           =    6.57
    WITHIN VARIANCE      =    1.09
    F (H:BNO)            =    3.13
```

```
REGRESSION ANALYSIS OF TEST TIRE CALIBRATION DATA
        (RUN DATA  VS  RUN ORDER)

    TIRE          :  E501
    TEST SERIES   :  40-2
    TIME          :  PM
    REPLICATION   :  4
    CONDITION     :  NEW
    SPEED         :  60
    SURFACE       :   6
    WATER DEPTH   :  .033
    ROAD TEMP     :   61 F
    GROOVE DEPTH  :  0.364 IN.

STATISTICAL ANALYSIS :
---------------------

    MEAN SKID RESISTANCE =   29.80
    VARIANCE             =    5.71
    STD. DEV.            =    2.39
    NUMBIJ  SS(X,Y)      =   23.00
    DENBIJ  SS(X,X)      =   42.00
    BIJ                  =    0.55
    BIJ X NUMBIJ         =   12.60
    ERROR S.S.           =   27.40
    WITHIN VARIANCE      =    4.57
    F (H:BNO)            =    2.76
```

Figure D-1. Order-of-Run analysis output.

MEAN SKID RESISTANCE	- $\sum_{i=1}^{n} SN_i/n = \overline{SN}$
VARIANCE	- $\left[\sum_{i=1}^{n} SN_i^2 - n\,\overline{SN}^2\right]/(n-1) = s^2$
STD. DEV.	- $\sqrt{s^2}$
NUMBIJ	- $\sum_{i=1}^{n} SN_iX_i$ (SEE NOTE 1)
DENBIJ	- $\sum_{i=1}^{n} X_iX_i = 42$
BIJ (SLOPE COEFFICIENT)	- $\sum_{i=1}^{n} SN_iX_i \Big/ \sum_{i=1}^{n} X_iX_i$
BIJ x NUMBIJ	- $BIJ \sum_{i=1}^{n} SN_iX_i$
ERROR S.S.	- $\sum_{i=1}^{n} SN_i^2 - n\,\overline{SN}^2 - BIJ \sum_{i=1}^{n} SN_iX_i$
WITHIN VARIANCE	- ERROR S.S./(n - 2)
F (H: BIJ $\neq$ 0) $F_{1,6} = 6.0$ (α=.95)	- $BIJ \sum_{i=1}^{n} SN_iX_i$ / WITHIN VARIANCE

note 1: RUN ORDER (X_i) WAS ADJUSTED FOR THE MEAN TO SIMPLIFY ANALYSIS i.e., $X_i = -3.5, -2.5, -1.5, -.5, .5, 1.5, 2.5, 3.5,$

HENCE $\sum_{i=1}^{n} X_i = 0.0$

Figure D-2. Formulas used in Statistical Analysis

The second step in the data analysis involved evaluation of the mean data for treatment effects using the methods of analyses of variance and covariance. Two programs from the Biomedical Computer Programs Package[1], BMD02V (Analysis of Variance for Factorial Design), and BMD03V (Analysis of Covariance for Factorial Design) were used for this phase of the analysis. The selection of these programs was based more on availability than on past experience. Hence, this phase not only provided the desired analyses, but also afforded the authors a chance to evaluate the two programs as to their merits and demerits. The original analysis called for an analysis of covariance by tire for each set and a combined analysis for all four sets (set 2 was not used for this phase of the analysis). Figure D-3 shows a typical output from the Analysis of Covariance program. This particular run is for all four sets of data (384 points), for the E 249 tire with three (3) analysis of variance classifications and two covariates (groove depth and pavement temperature). The model is given below:

$$SN_{14} = \mu + H_i + P_j + V_k + (HP)_{ij} + (HV)_{ik} + (PV)_{jk} + (HPV)_{ijk} + \beta_1 x_1 + \beta_2 x_2 + e_{ijk}$$

H - Water level
P - Pavement
V - Speed
x_1 - Groove Depth
x_2 - Pavement Temperature

The variable formatting feature for data input greatly enhances the flexibility of these programs and was particularly useful during this study. Unfortunately, some of the output is difficult to interpret. The intermediate results, i.e, Tables of Variation, for each variable are not fully explained in the documentation and interpretation to the unfamiliar user is extremely difficult. Also, the meaning and derivation of "REGRESSION COEFFICIENTS," at each step is not clearly detailed in the program documentation. The COMPUTED T VALUES and the F-STATISTIC, used to test hypotheses about the regression coefficients are explained and were used in this analysis. Given that β is the covariate coefficient vector $(\beta_1, \beta_2, \ldots, \beta_p)$, T is used to test the hypothesis $\beta_i = 0$, and F is used to test the hypothesis that $\beta = \beta_0$, where β_0 is a constant. A thorough examination of the theory of covariance analysis involving two covariates is contained in Appendix C.

(1) Biomedical Computer Programs, UCLA, 1970 Version

```
BMD03V - ANALYSIS OF COVARIANCE - REVISED JANUARY 29, 1970
HEALTH SCIENCES COMPUTING FACILITY, UCLA


PROBLEM NO.  E249

NO. OF VARIABLES     3
NO. OF REPLICATES   16
NO. OF COVARIATES    2

VARIABLE      NO. OF LEVELS
    1              2
    2              4
    3              3



VARIABLE FORMAT
(F9.4,F9.4,2X,F3.0)



TABLE OF VARIATION FOR VARIABLE  1
VARIATE
SS  140.58965       -0.35336     2577.62451
COVARIATES
                     0.00089       -6.47878
                                 47259.35547


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
    1692.02734      36.11871     1575.91382
COVARIATES
                     4.11280      -34.79230
                                 77854.06250


INVERSE OF COVARIATES MATRIX
                     0.24407        0.00011
                                    0.00001

REGRESSION COEFFICIENTS
                 8.98725       0.02426


TABLE OF VARIATION FOR VARIABLE  2
VARIATE
   42517.75781     -13.66798   -11776.12891
COVARIATES
                     0.00476        4.00665
                                  3667.34204


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
   44069.19531      22.80408   -12777.83594
COVARIATES
```

Figure D-3. Biomed. analysis of covariance output.

```
                    4,11667      -24.30687
                                34262.08984
INVERSE OF COVARIATES MATRIX
                    0.24394        0.00017
                                   0.00003

REGRESSION COEFFICIENTS
                 3.35125      -0.37057


TABLE OF VARIATION FOR VARIABLE  3
VARIATE
   8751.03125      -0.41811      -793.34326
COVARIATES
                    0.00004        0.01479
                                  95.69269


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
  10302.46875      36.05396     -1795.05396
COVARIATES
                    4.11195       -28.29872
                                30690.44141


INVERSE OF COVARIATES MATRIX
                    0.24475        0.00023
                                   0.00003

REGRESSION COEFFICIENTS
                 8.41894      -0.05073


TABLE OF VARIATION FOR VARIABLE  1 2
VARIATE
     93.17188      -0.15807      -102.24219
COVARIATES
                    0.00065        0.27604
                                 155.44531


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
   1644.60962      36.31400     -1103.95288
COVARIATES
                    4.11256       -28.03748
                                30750.19531


INVERSE OF COVARIATES MATRIX
                    0.24468        0.00022
                                   0.00003

REGRESSION COEFFICIENTS
                 8.63895      -0.02802


TABLE OF VARIATION FOR VARIABLE  1 3
VARIATE
     22.52344       0.00798       -14.87598
```

Figure D-3. (continued).

```
COVARIATES
                              0.00001       0.00298
                                           26.06250

TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
    1573.96118        36.48004    -1016.58667
COVARIATES
                       4.11192      -28.31053
                                 30620.81250


INVERSE OF COVARIATES MATRIX
                       0.24475        0.00023
                                      0.00003

REGRESSION COEFFICIENTS
                     8.69856      -0.02516



TABLE OF VARIATION FOR VARIABLE  2 3
VARIATE
     771.05469         0.00349      -19.53516
COVARIATES
                       0.00000        0.00403
                                     10.19971


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
    2322.49243        36.47556    -1021.24585
COVARIATES
                       4.11192      -28.30949
                                 30604.94922


INVERSE OF COVARIATES MATRIX
                       0.24475        0.00023
                                      0.00003

REGRESSION COEFFICIENTS
                     8.69631      -0.02532



TABLE OF VARIATION FOR VARIABLE  1 2 3
VARIATE
      81.35156        -0.01240       14.35156
COVARIATES
                       0.00000        0.00581
                                     19.96875


TABLE OF VARIATION ADJUSTED BY RESIDUALS
VARIATE
    1632.78931        36.45957     -987.35913
COVARIATES
                       4.11191      -28.30771
                                 30614.71875


INVERSE OF COVARIATES MATRIX
                       0.24475        0.00023
                                      0.00003
```

Figure D-3. (continued).

```
REGRESSION COEFFICIENTS
                    8.70019      -0.02421


TABLE OF VARIATION FOR RESIDUALS
VARIATE
   1551.43774       36.47208    -1001.71069
COVARIATES
                     4.11191      -28.31352
                                30594.75000


INVERSE OF COVARIATES MATRIX
                     0.24476        0.00023
                                    0.00003

REGRESSION COEFFICIENTS
                    8.69982      -0.02469


COMPUTED T VALUES
                    9.56752      -2.34214

RESIDUAL MEAN SQUARE             3.37823
SQRT. OF RESID. MEAN SQ.         1.83800

F % 2, 358< #      50.62297


TABLE OF VARIATION FOR TOTAL
VARIATE
   53928.91797      21.87361   -11115.85547
COVARIATES
                     4.11827      -30.48199
                                81828.81250
```

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUMS OF SQUARES	MEAN SQUARES
1	1	119.78589	119.78589
2	3	38048.33203	12682.77734
3	2	8698.46875	4349.23438
12	3	90.55298	30.18433
13	2	21.65869	10.82935
23	6	770.02222	128.33704
123	6	82.27783	13.71297
WITHIN REPLICATES	358	1209.40503	3.37823

Figure D-3. (continued).

The analysis of variance output is shown in Figure D-4. A complete set of these outputs is given in Appendix E. This particular output summarizes a 2 x 2 x 4 x 3 design using tire condition (two levels), water depth (two levels), surface (four levels), and speed (three levels). Once the levels (treatments) are identified, the analysis of variance table can be used to construct F-statistics for hypothesis testing on each source of variation. Following the ANOVA table, the cell means are given along with marginal means by variable number. This analysis of variance was performed by tire for each set of data, along with a combined analysis for sets 1, 3, 4, 5.

The third, and final, step in the analysis procedure was the evaluation of various calibration (prediction) equations. This phase of the analysis employed the OMNITAB Computing System developed by the National Bureau of Standards(2). The OMNITAB program is a "user-oriented" system which features an easily learned instruction set and an internally stored (in core memory) data matrix (worksheet) with a maximum capacity of 12,500 data points. The worksheet consists of columns (rows) of data which can be manipulated, analyzed, and plotted, etc. using a simple command language. The program also provides for a variety of input media (card, tape, disk). Using this program, a mass of different regression equations were evaluated. Discussion of the results of the calibration analysis is included in the body of this report. It is sufficient to note here, that the use of the OMNITAB system made possible the evaluation of many calibrations with relatively little effort using the standard output of the OMNITAB FIT command.

The FIT command will perform a regression analysis and produce a four-page standard output of results. Figure D-5 shows one such output for the model:

$$SNY = \beta_1 SNX + \beta_2 D + \beta_3 T$$

where: $D = G_{249} - G_{501}$ (tire groove depth difference, inches)

$T = T_{249} - T_{501}$ (temperature difference, deg. F)

The OMNITAB program for this example is also shown to demonstrate the simplicity of the input instruction set.

Page 1 of the standard output summarizes the input (predictor variables) and output (predicted values, residuals, etc.) for each observation (designated as row number). Page 2

(2) The OMNITAB Computing System, National Bureau of Standards, 1971

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BMD02V - ANALYSIS OF VARIANCE FOR FACTORIAL DESIGN - REVISED SEPTEMBER 12, 19(
HEALTH SCIENCES COMPUTING FACILITY, UCLA
```

ANOVA, SETS 1,5,3 & 4

TIRE 249

```
PROBLEM NO. 14

NUMBER OF VARIABLES      4
NUMBER OF REPLICATES     8

VARIABLE      NO. OF LEVELS
    1              2
    2              2
    3              4
    4              3

GRAND MEAN           27.26529
```

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUMS OF SQUARES	MEAN SQUARES
1	1	16.71819	16.71819
2	1	140.59018	140.59018
3	3	42517.90625	14172.63281
4	2	8751.05164	4375.52344
12	1	2.17133	2.17133
13	3	12.59440	4.19813
14	2	0.85527	0.42764
23	3	93.22526	31.07507
24	2	22.52182	11.26091
34	6	771.13040	128.52173
123	3	1.27035	0.42345
124	2	0.80035	0.40018
134	6	6.18634	1.03106
234	6	81.59929	13.59988
1234	6	6.74219	1.12370
WITHIN REPLICATES	336	1503.62012	4.47506
TOTAL	383	53928.95703	

Figure D-4. Biomed. analysis of variance output.

PROBLEM NO. 14

CELL NUMBERS				MEANS
1	1	1	1	26.63748
1	1	1	2	16.00780
1	1	1	3	11.23593
1	1	2	1	22.61404
1	1	2	2	16.47968
1	1	2	3	13.17031
1	1	3	1	50.19519
1	1	3	2	40.22809
1	1	3	3	35.74841
1	1	4	1	39.89996
1	1	4	2	33.01559
1	1	4	3	32.62030
1	2	1	1	23.16405
1	2	1	2	15.08593
1	2	1	3	11.36718
1	2	2	1	22.04686
1	2	2	2	16.77499
1	2	2	3	13.06093
1	2	3	1	50.31241
1	2	3	2	39.63120
1	2	3	3	33.18591
1	2	4	1	36.87805
1	2	4	2	30.97029
1	2	4	3	29.04842
2	1	1	1	26.23123
2	1	1	2	15.07656
2	1	1	3	11.22031
2	1	2	1	21.46873
2	1	2	2	15.80781
2	1	2	3	13.38750
2	1	3	1	49.79214
2	1	3	2	40.18903
2	1	3	3	35.55466
2	1	4	1	39.43277
2	1	4	2	32.00311
2	1	4	3	30.87654
2	2	1	1	23.66405
2	2	1	2	15.01562
2	2	1	3	11.65156
2	2	2	1	21.21873
2	2	2	2	16.54999
2	2	2	3	12.80312
2	2	3	1	50.38275
2	2	3	2	39.92181
2	2	3	3	32.89059
2	2	4	1	35.20154
2	2	4	2	30.52029
2	2	4	3	28.50311

MARGINAL MEANS

VARIABLES	CATEGORIES	MEANS
1	1	27.47412
	2	27.05681
2	1	27.87054
	2	26.66039
3	1	17.19646
	2	17.11520
	3	41.50266
	4	33.24747
4	1	33.69624
	2	25.82985
	3	22.27029

Figure D-4. (continued).

OMNITAB CALIBRATION MODEL SN14 VS SN15 ALL 384 PTS WITHOUT MEAN


```
                LEAST SQUARES FIT FOR        DATA       IN COLUMN    1
       AS A LINEAR FUNCTION OF  3 PREDICTOR VARIABLES  IN COLUMNS   4,   2,   8
  USING  384 NON-ZERO WEIGHTS AND    0 ZERO WEIGHTS   IN COLUMN    7
```

ROW	PREDICTOR VARIABLES IN COL. 4	COL. 2	COL. 8	DATA COL. 1	PREDICTED VALUES	STD. DEV. OF PRED. VALUES	RESIDUALS	STD. RES.	WEIGHTS
1	49.19	-.03200	1.000	50.099899	47.379929	.14722008	2.7199564	1.52	1.000
2	48.54	-.02400	1.000	49.687393	46.893265	.15047443	2.7941198	1.57	1.000
3	51.14	-.02500	0.	52.737488	49.481674	.15815461	3.2558079	1.83	1.000
4	50.62	.002000	-2.000	50.237396	49.616028	.22228241	.62135655	0.35	1.000
5	49.55	-.005000	1.000	49.562500	48.234161	.18958014	1.3283377	0.75	1.000
6	51.02	-.008000	1.000	49.299988	49.619064	.19827456	-.31908160	-0.18	1.000
7	47.63	-.006000	1.000	49.699997	46.298508	.17467892	3.4014807	1.91	1.000
8	51.19	-.02900	-3.000	52.449997	49.658691	.18209410	2.7913017	1.57	1.000
9	50.50	-.03200	1.000	51.037399	48.661804	.15133905	2.3755915	1.33	1.000
10	50.11	-.03200	2.000	49.574997	48.215897	.15596020	1.3590956	0.76	1.000
11	49.55	-.04200	-5.000	49.824997	47.953629	.21425742	1.8713598	1.05	1.000
12	50.63	-.03500	-4.000	51.274867	49.065765	.19539994	2.2091179	1.24	1.000
13	52.46	-.03700	2.000	50.487488	50.418442	.16192997	.069044709	0.04	1.000
14	47.84	-.02000	1.000	49.037399	46.283676	.15288198	2.7537174	1.54	1.000
15	50.36	-.04400	1.000	43.337494	48.268326	.15597826	-4.9308348	-2.77	1.000
16	52.21	-.03900	-1.000	51.549896	50.339630	.16015875	1.2102528	0.68	1.000
17	51.94	.007000	1.000	49.574997	50.788118	.23209780	-1.2131262	-0.68	1.000
18	49.44	.01200	0.	49.437393	48.506622	.23677135	.93076289	0.52	1.000
19	48.01	.008000	-5.000	51.012390	47.378082	.26645046	3.6343050	2.05	1.000
20	52.56	.004000	1.000	51.612396	51.342957	.22589910	.26942796	0.15	1.000
21	51.94	-.05100	0.	52.212494	49.781311	.16469944	2.4311186	1.36	1.000
22	49.94	-.04100	0.	49.299988	48.013275	.15028557	1.2867022	0.72	1.000
23	48.39	.01400	3.000	48.649994	47.315750	.25012600	1.3342342	0.75	1.000
24	51.94	-.04300	4.000	49.574997	49.659637	.18645716	-.084649980	-0.05	1.000
25	49.44	.01200	0.	49.437500	48.506516	.23677087	.93097752	0.52	1.000
26	48.39	-.02300	3.000	47.899994	46.630447	.16752946	1.2695389	0.71	1.000
27	49.46	-.04800	-1.000	49.137500	47.437167	.16065508	1.7003241	0.95	1.000
28	52.20	-.04200	1.000	50.349991	50.136932	.15732641	.21304762	0.12	1.000
29	52.81	-.04900	-1.000	52.787399	50.740433	.16336065	2.0469637	1.15	1.000
30	52.45	-.04000	-1.000	51.137497	50.553177	.16107208	.58431405	0.33	1.000
31	48.06	-.003000	-2.000	50.987488	47.020767	.19947213	3.9667120	2.23	1.000
32	50.74	-.06500	-4.000	52.399887	48.619980	.22774760	3.7799053	2.13	1.000
33	30.30	-.04600	4.000	27.549988	28.471375	.17035621	-.92139119	-0.52	1.000
34	30.30	-.04000	-4.000	30.174988	29.122208	.16776007	1.0527716	0.59	1.000
35	28.60	-.03600	0.	28.174988	27.461426	.10595000	.71355122	0.40	1.000
36	26.27	-.03800	1.000	24.549988	24.890839	.11203021	-.34085250	-0.19	1.000
37	26.92	-.03900	-3.000	23.212494	25.776993	.14521331	-2.5645037	-1.44	1.000
38	25.57	-.03700	3.000	22.424988	24.090744	.13312638	-1.6657686	-0.93	1.000
39	26.19	-.01700	0.	25.524994	27.215134	.084551692	-1.6901541	-0.95	1.000
40	30.42	-.03800	-4.000	22.924988	29.281342	.16447574	-6.3563565	-3.57	1.000
41	31.82	.001000	0.	29.174988	31.101196	.13254201	-1.9262114	-1.08	1.000
42	22.46	-.01800	0.	23.099991	21.605148	.067342401	1.4948349	0.84	1.000
43	30.80	-.04400	-6.000	31.174988	29.671402	.21862471	1.5035801	0.85	1.000
44	29.30	.02600	0.	24.799988	29.098114	.20443499	-4.2981339	-2.42	1.000
45	27.34	-.003000	-1.000	25.324997	26.711746	.10909569	-1.3867493	-0.78	1.000
46	28.16	-.002000	0.	26.112488	27.468552	.10957885	-1.3560696	-0.76	1.000
47	30.05	-.003000	0.	28.549988	29.293503	.11473680	-.74351889	-0.42	1.000
48	32.05	.02500	0.	30.174988	31.765457	.21135557	-1.5904760	-0.89	1.000
49	25.20	.01000	2.000	22.949997	24.662491	.14265269	-1.7124958	-0.96	1.000

Figure D-5. Omnitab output from regression analysis.

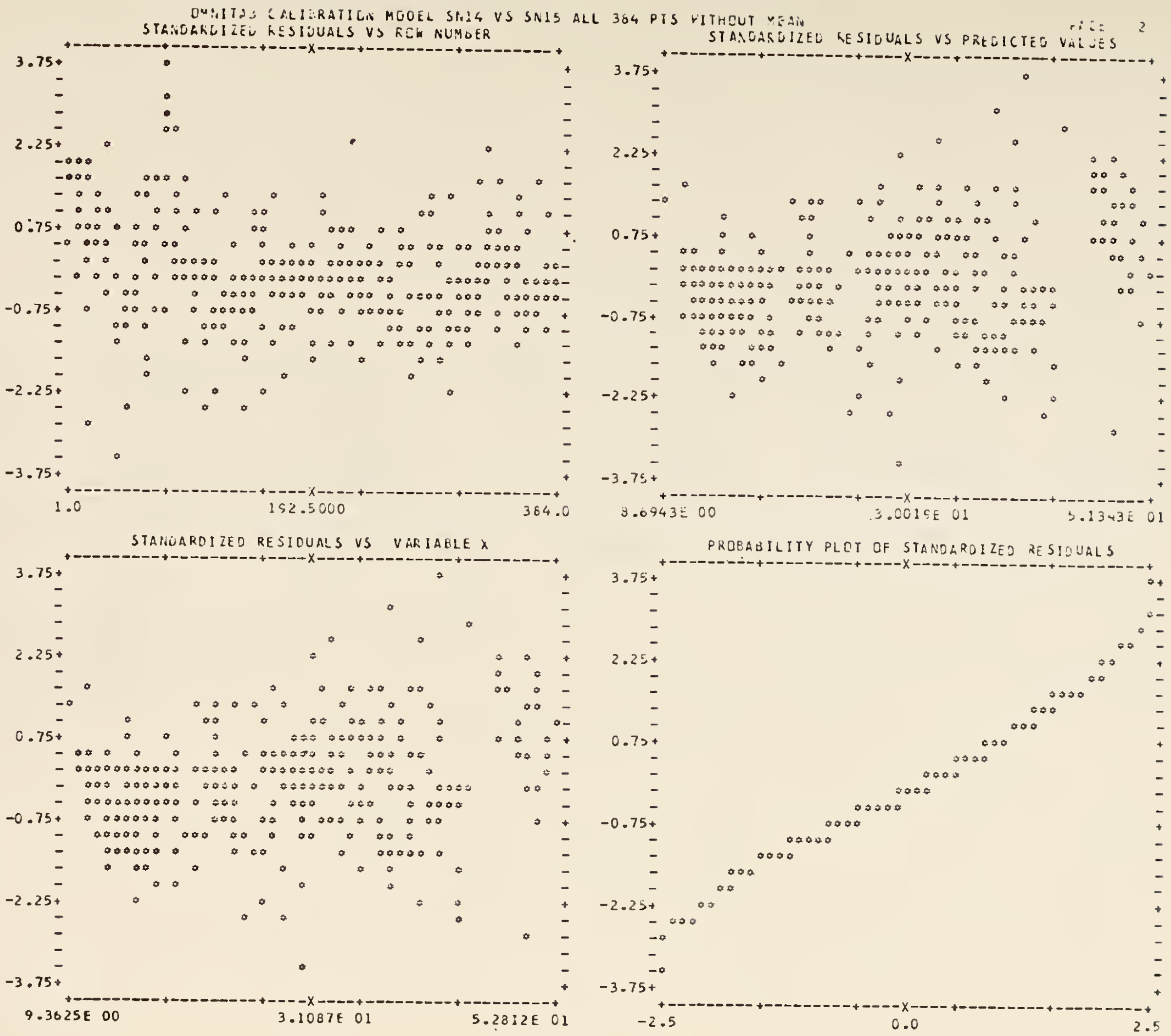

Figure D-5. (continued).

OMNITAB CALIBRATION MODEL SN14 VS SN15 ALL 384 PTS WITHOUT MEAN PAGE 3

LEAST SQUARES FIT FOR DATA IN COLUMN 1
AS A LINEAR FUNCTION OF 3 PREDICTOR VARIABLES IN COLUMNS 4, 2, 8
USING 384 NON-ZERO WEIGHTS AND 0 ZERO WEIGHTS IN COLUMN 7

VARIANCE-COVARIANCE MATRIX OF THE ESTIMATED COEFFICIENTS

COLUMN	4	2	8
4	1.6633261-05		
2	.011076022	15.737926	
8	-6.1342553-06	3.5154354-04	8.0880057-04

ANALYSIS OF VARIANCE
-DEPENDENT ON ORDER VARIABLES ARE ENTERED, UNLESS VECTORS ARE ORTHOGONAL-

COLUMN	SS=RED. DUE TO COEF.	CUM. MS REDUCTION	D.F.	CUM. RESIDUAL MS	D.F.	F(COEF=0)	P(F)	F(COEFS=0)	P(F)
4	338088.81	338088.61	1	3.4162807	383	105551.500	0.000	35192.988	0.000
2	70.042404	169079.38	2	3.2418671	382	21.867	0.000	13.747	0.000
8	18.024277	112725.56	3	3.2030678	381	5.627	0.018	5.627	0.018
RESIDUAL	1220.3691		381						
TOTAL	339397.69		384						

Figure D-5. (continued).

OMNITAS CALIBRATION MODEL SN14 VS SN15 ALL 384 PTS WITHOUT MEAN PAGE 4

LEAST SQUARES FIT FOR DATA IN COLUMN 1
AS A LINEAR FUNCTION OF 3 PREDICTOR VARIABLES IN COLUMNS 4, 2, 8
USING 384 NON-ZERO WEIGHTS AND 0 ZERO WEIGHTS IN COLUMN 7

	ESTIMATES FROM LEAST SQUARES FIT				FIT OMITTING LAST COLUMN		
COLUMN	COEFFICIENT	S.D. OF COEFF.	RATIO	*ACC. DIGITS	COEFFICIENT	S.D. OF COEFF.	RATIO
4	.97667509	.0040783682	239.48	4.23	.97616339	.0040972754	238.25
2	18.521744	3.9671049	4.67	3.50	18.551071	3.9910431	4.65
6	-.067463160	.028439425	-2.37	3.44			
RESIDUAL STANDARD DEVIATION =			1.7897120			1.8005199	
BASED ON DEGREES OF FREEDOM			384- 3 = 381			384- 2 = 382	

* THE NUMBER OF CORRECTLY-COMPUTED DIGITS IN EACH COEFFICIENT USUALLY DIFFERS BY LESS THAN 1 FROM THE NUMBER GIVEN HERE

Figure D-5. (continued).

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OMNITAB CALIBRATION MODEL SN14 VS SN15 ALL 384 PTS WITHOUT MEAN                    PAGE  13

LIST OF COMMANDS, DATA AND DIAGNOSTICS

    DIMENSION 384 BY 10
    FORMAT A(2F9.4,2X,F3.0)
    RESET 384
    ADD 0.0,1.0,7
    1/ SUBTRACT 6,3,8
    2/ SUBTRACT 5,2,2
    3/ RESET 384
    4/ FIT 1,7,3,4,2,8,9 10
    4.5/ FIT 1,7,2,4,2,9,10
    5/ FIT 1,7,1,4,9,10
    READ A 384 1***3
      384 DATA CARD(S) READ BUT NOT LISTED
    READ A 384 4***6
      384 DATA CARD(S) READ BUT NOT LISTED
    PERFORM 1,5
    STOP

          NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.  20234, OMNITAB II VERSION 5.00 MAY 15, 1971
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Figure D-5. (continued).

of the standard output contains four plots of the standardized residuals. The plot in the upper left plots the standardized residuals against the run order (row number) in an attempt to identify time trends in the data. The plot in the upper right uses the predicted values as the abscissa to identify possibly a non-randomness indicating non-constant variance or that some important variable(s) has been excluded from the model. The third plot (lower left) uses the predictor (independent) variable as the abscissa. This plot has more meaning for the polynomial fit command, but may or may not have much meaning for the FIT depending on the order and character of the predictor variables. The last graph gives a probability plot of the standardized residuals. The plot is meant to give a rough graphic measure of how well the statistical model fits the data (the points should lie approximately on a straight line). A more detailed discussion of this and the other three plots can be found in NBS Technical Note 552, (OMNITAB) II User's Reference Manual, available from the U.S. Government Printing Office.

Page 3 lists the variance-covariance matrix and an analysis of variance table for the coefficients with F-tests to measure significance. The fourth page is divided into three parts: (a) estimates, (b) accuracy, and (c) estimates from a refit omitting the last term (variables should be entered into the regression so that the least significant variable is last). A complete discussion of the output on pages 3 and 4 is beyond the scope of this discussion, however, a detailed explanation can be found in NBS Technical Note 552.

The OMNITAB program was also used to perform an analysis of covariance by tire for the combined data (sets 1, 3, 4, 5). The model chosen is the same as that used for the Biomed Covariance Analysis (Eq. D-1). However, in this case, the third order interaction term $(HPV)_{ijk}$ is dropped, simplifying the calculations. Hence, the contribution of this term (though small) is "absorbed" by other terms in the model.

The first step in building the covariance model is the generation of a design matrix in the OMNITAB worksheet. A part of this design matrix is shown in figure D-6. Each element in the matrix indicates the presence (1) or absence (0) of the effect of the i th level of the factor (water depth, surface, velocity). For instance, in row 1, column 3, the 1 indicates the presence of the effect of the first level of factor P, i.e., data was taken on pavement 1.

WATER	SURFACE	SPEED	FACTOR	MEAN	H	P			V		HP			HV		PV					
			DEGREES OF FREEDOM	1	1	3			2		3			2		6					
i	j	k	PARAMETERS	μ	h_1	p_1	p_2	p_3	v_1	v_2	$(hp)_{11}$	$(hp)_{12}$	$(hp)_{13}$	$(hv)_{11}$	$(hv)_{12}$	$(pv)_{11}$	$(pv)_{12}$	$(pv)_{21}$	$(pv)_{22}$	$(pv)_{31}$	$(pv)_{32}$
1	1	1		1	1	1	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0
1	1	2		1	1	1	0	0	0	1	1	0	0	0	1	0	1	0	0	0	0
1	1	3		1	1	1	0	0	-1	-1	1	0	0	-1	-1	-1	-1	0	0	0	0
1	2	1		1	1	0	1	0	1	0	0	1	0	1	0	0	0	1	0	0	0
1	2	2		1	1	0	1	0	0	1	0	1	0	0	1	0	0	0	1	0	0
1	2	3		1	1	0	1	0	-1	-1	0	1	0	-1	-1	0	0	-1	-1	0	0
1	3	1		1	1	0	0	1	1	0	0	0	1	1	0	0	0	0	0	1	0
1	3	2		1	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	1
1	3	3		1	1	0	0	1	1	1	0	0	1	-1	-1	0	0	0	0	-1	-1
1	4	1		1	1	-1	-1	-1	1	0	-1	-1	-1	1	0	-1	0	-1	0	-1	0
1	4	2		1	1	-1	-1	-1	0	1	-1	-1	-1	0	1	0	-1	0	-1	0	-1
1	4	3		1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
2	1	1		1	-1	1	0	0	1	0	-1	0	0	-1	0	1	0	0	0	0	0
2	1	2		1	-1	1	0	0	0	1	-1	0	0	0	-1	0	1	0	0	0	0
2	1	3		1	-1	1	0	0	-1	-1	-1	0	0	1	1	-1	-1	0	0	0	0
2	2	1		1	-1	0	1	0	1	0	0	-1	0	-1	0	0	0	1	0	0	0
2	2	2		1	-1	0	1	0	0	1	0	-1	0	0	-1	0	0	0	1	0	0
2	2	3		1	-1	0	1	0	-1	-1	0	-1	0	1	1	0	0	-1	-1	0	0
2	3	1		1	-1	0	0	1	1	0	0	0	-1	-1	0	0	0	0	0	1	0
2	3	2		1	-1	0	0	1	0	1	0	0	-1	0	-1	0	0	0	0	0	1
2	3	3		1	-1	0	0	1	-1	-1	0	0	-1	1	1	0	0	0	0	-1	-1
2	4	1		1	-1	-1	-1	-1	1	0	1	1	1	-1	0	-1	0	-1	0	-1	0
2	4	2		1	-1	-1	-1	-1	0	1	1	1	1	0	-1	0	-1	0	-1	0	-1
2	4	3		1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1

Figure D-6. Design matrix for Ominitab covariance model.

Because there are four replications per set, for each of 4 sets, there are effectively 16 replications of the data for this analysis. The design matrix in figure D-6 represents one replication (2 water depths x 4 surfaces x 3 speeds = 24). The entire design matrix then consists of 24 x 16 = 384 rows. This design matrix is generated in the worksheet using various manipulative commands, and then the FIT command is used to produce the output shown in figure D-7.

In this example, the design matrix was stored in columns 8 through 24, column 7 contained 1 to fit the mean, and columns 2 and 3 contained the covariate data, pavement temperature, and groove depth, respectively. In figure D-7, page 1 of the standard output (format previously described) is omitted due to its length. Page 2 shows the various plots of the standardized residuals. Note here, however, the third plot (lower left) has no meaning because the abscissa is a column of + 1 (column 8 of the design matrix). Pages 3 and 4 of the output give the ANOVA table and coefficient estimates from the least squares fit. The coefficients listed on page 4 do not, unfortunately, thoroughly represent the entire model. For instance, only three coefficients are given for the effects of factor P, i.e., p_1, p_2, p_3. The missing coefficients can be computed as linear combinations of those given, and in this case $p_4 = -(p_1 + p_2 + p_3)$. Similarly, one can derive the appropriate linear combination to estimate the remaining missing coefficients.

Although the foregoing is a fairly complicated example, the construction of the analysis of covariance model using OMNITAB is not difficult. The additional effort expended over using a "canned program" such as BMD03V is more than compensated for by the additional utility of a programmable language such as OMNITAB. For example, the "worksheet" feature of OMNITAB allows storage of intermediate results and a continued analysis of the data in the same job - a feature lacking in most statistical utility programs.

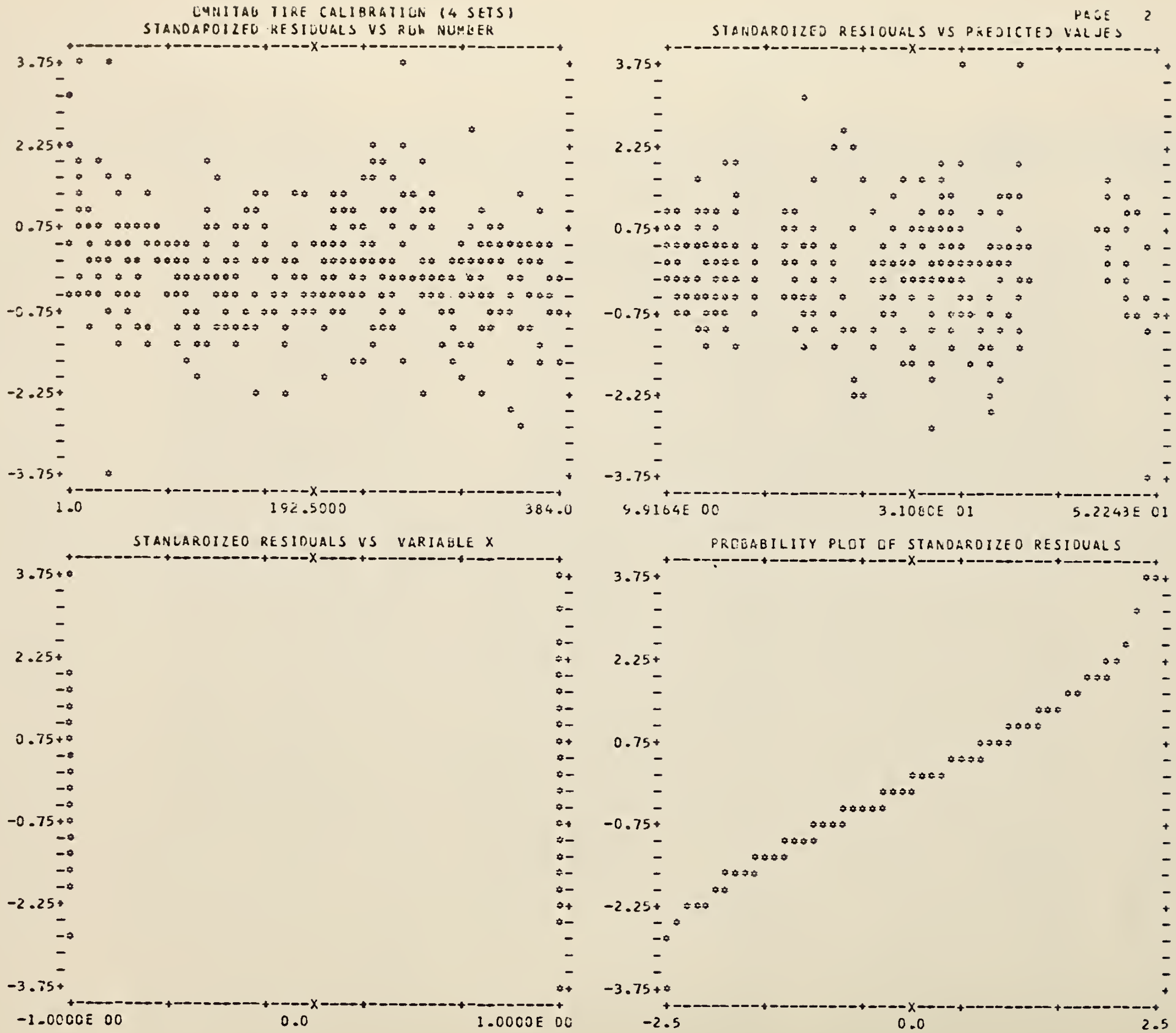

Figure D-7. Omnitab output for covariance output.

LEAST SQUARES FIT FOR DATA IN COLUMN 1
AS A LINEAR FUNCTION OF 20 PREDICTOR VARIABLES IN COLUMNS 7, 8, 9, 10, 11, 12, 13, 14,
15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 3, 2
USING 384 NON-ZERO WEIGHTS AND 0 ZERO WEIGHTS IN COLUMN 25

VARIANCE-COVARIANCE MATRIX OF THE ESTIMATED COEFFICIENTS

COLUMN	7	8	9	10	11	12	13
7	.71596748						
8	.099216223	.024695128					
9	.038207039	.0055236891	.031221021				
10	.016839143	.0027245439	-.0091070980	.030428644			
11	-.030703556	-.0046016052	-.011237755	-.010736465	.030430302		
12	-.0062403344	-.0010327296	-2.2452400-04	-5.0941203-04	3.9614225-04	.019476838	
13	.0032402086	-6.9216266-04	.0016614345	-.0040769763	.0011852566	-.0082968846	.020910852
14	.0061499365	7.6033702-04	4.6744221-04	-2.1610641-04	-1.5422807-04	6.5914266-05	.0014381961
15	.0044261441	9.3039707-04	-1.4466190-04	.0011870211	-5.3935731-04	-3.9913575-04	-.0041612014
16	-.0013592169	-2.6552356-04	6.1935207-05	-3.9161090-04	1.6950776-04	1.3057536-04	.0013893067
17	-.0032942058	-5.9596379-04	-5.8509482-05	-4.3956935-04	2.6422809-04	1.5039837-04	.0013735266
18	.0013736838	1.0875252-04	2.0785243-04	-3.2829377-04	5.2861258-05	1.0737414-04	.0014067655
19	-4.7203177-04	3.3465723-05	-1.5481054-04	3.5008020-04	-9.4824150-05	-1.1600539-04	-.0014000309
20	6.3749915-04	1.9319422-04	-9.3520139-05	3.7615746-04	-1.4381693-04	-1.2561956-04	-.0013925307
21	.0015324065	-7.4457203-05	4.7324994-04	-.0010470713	2.7784845-04	3.4649321-04	.0042013563
22	.0018000316	-4.1123538-05	4.8666913-04	-.0010410217	2.6693463-04	3.4439703-04	.0042026762
23	-4.3664337-04	3.4539305-05	-1.5390372-04	3.5072326-04	-9.5632509-05	-1.1609468-04	-.0013999997
24	-.0014058554	-1.0412394-04	-2.0720805-04	3.2797675-04	-5.3165655-05	-1.0757596-04	-.0014064955
3	-.0089801624	-.0013482037	-5.1046000-04	-2.1370212-04	4.0522264-04	8.2368904-05	-6.3552230-05
2	-.26130652	-.0079375729	-.0066594332	-.0047255866	.0059994919	7.7730534-04	-2.1567986-04

COLUMN	14	15	16	17	18	19	20
14	.029127087						
15	-.0099760075	.030065361					
16	-.0095365368	-.010021232	.029073954				
17	9.2579241-05	-3.8228590-04	1.2609105-04	.019436188			
18	1.3509238-04	-3.5562133-04	1.1935162-04	-.0095357783	.019424632		
19	-1.2675716-04	3.6074966-04	-1.2047510-04	-1.1857605-04	-1.2273517-04	.058021009	
20	-1.1680744-04	3.6711502-04	-1.2230519-04	-1.2368616-04	-1.2029937-04	-.028829613	.058020871
21	3.8149441-04	-.0010616067	3.6150008-04	3.5493542-04	3.6863377-04	-.019665543	.0092870854
22	3.8377172-04	-.0010800548	3.6091381-04	3.5386626-04	3.6911690-04	.0092838148	-.019662267
23	-1.2651052-04	3.6096130-04	-1.2065063-04	-1.1862782-04	-1.2272902-04	-.019177813	.0097708330
24	-1.3516767-04	3.5540177-04	-1.1902626-04	-1.1419666-04	-1.2484495-04	.0097725764	-.019179430
3	-8.1625578-05	-5.1330120-05	1.3185384-05	4.2865469-05	-2.0770312-05	7.9479933-06	-6.5104066-06
2	-.0017080100	-.0016084453	.0012913716	3.4592976-04	2.0374733-05	8.6461218-05	-2.0817127-04

COLUMN	21	22	23	24	3	2
21	.058997203					
22	-.027850874	.058998581				
23	-.019665573	.0092838461	.058021009			
24	.0092813522	-.019668747	-.028826874	.058023922		
3	-2.6154434-05	-2.9192626-05	7.8342837-06	2.0369356-05	1.2181132-04	
2	-3.7288439-05	-2.5533000-04	-3.4329350-05	2.4921959-04	8.3852885-04	.90691131

Figure D-7. (continued).

ANALYSIS OF VARIANCE
-DEPENDENT ON ORDER VARIABLES ARE ENTERED, UNLESS VECTORS ARE ORTHOGONAL-

COLUMN	SS=RED. DUE TO COEF.	CUM. MS REDUCTION	D.F.	CUM. RESIDUAL MS	D.F.	F(COEF=0)	P(F)	F(COEFS=0)	P(F)
7	285467.88	285467.88	1	140.80806	383	77038.000	0.000	4561.387	0.000
8	140.59010	142804.19	2	140.80864	382	37.540	0.000	746.828	0.000
9	12366.486	99324.938	3	108.72025	381	3337.293	0.000	786.210	0.000
10	4206.0469	75545.189	4	97.937605	380	1135.068	0.000	636.147	0.000
11	25945.375	65625.250	5	29.738754	379	7001.770	0.000	604.964	0.000
12	8180.9531	56051.195	6	8.1747141	378	2207.760	0.000	178.510	0.000
13	477.62549	46112.113	7	6.9294863	377	128.895	0.000	33.564	0.000
14	35.127396	42102.492	8	6.8544922	376	9.480	0.002	26.231	0.000
15	148.73175	37440.957	9	6.4761534	375	40.138	0.000	27.627	0.000
16	.00022740182	33696.859	10	6.4934692	374	0.000	0.0	26.490	0.000
17	23.190491	30635.617	11	6.4487047	373	6.258	0.013	29.138	0.000
18	5.9370365	28083.141	12	6.4500809	372	1.602	0.206	31.681	0.000
19	284.59619	25944.785	13	5.7003603	371	76.603	0.000	35.441	0.000
20	1.8791914	24091.723	14	5.7106866	370	0.507	0.477	29.532	0.000
21	.027764579	22485.605	15	5.7260895	369	0.007	0.932	34.369	0.000
22	91.740128	21085.988	16	5.4923563	368	24.758	0.000	41.241	0.000
23	327.41626	19864.891	17	4.6151791	367	88.358	0.000	45.362	0.000
24	3.4275942	18761.473	18	4.6184244	366	0.925	0.337	31.030	0.000
3	32.893051	17775.758	19	4.5409603	365	8.877	0.003	46.083	0.000
2	308.63257	16902.402	20	3.7055435	364	83.289	0.000	83.289	0.000
RESIDUAL	1348.8181		364						
TOTAL	339397.69		364						

Figure D-7. (continued).

```
                LEAST SQUARES FIT FOR     DATA      IN COLUMN    1
       AS A LINEAR FUNCTION OF 20 PREDICTOR VARIABLES IN COLUMNS   7,   8,   9,  10,  11,  12,  13,  14,
15,  16,  17,  18,  19,  20,  21,  22,  23,  24,   3,   2
       USING  384 NON-ZERO WEIGHTS AND     0 ZERO WEIGHTS IN COLUMN   25
```

	ESTIMATES FROM LEAST SQUARES FIT				FIT OMITTING LAST COLUMN		
COLUMN	COEFFICIENT	S.D. OF COEFF.	RATIO	*ACC. DIGITS	COEFFICIENT	S.D. OF COEFF.	RATIO
7	26.970703	.84614867	31.87	4.24	29.474869	.88606888	33.26
8	.73501164	.15714681	4.68	3.61	.81107932	.17371660	4.67
9	-9.8391171	.17669463	-55.68	5.22	-9.7752981	.19544780	-50.01
10	-10.610202	.17443812	-60.83	4.70	-10.564915	.19302475	-54.73
11	14.343490	.17444283	82.22	4.87	14.286091	.19298267	74.03
12	6.5732403	.13955945	47.10	4.56	6.5657911	.15446964	42.50
13	-1.3934107	.14460568	-9.64	4.39	-1.3913441	.16007841	-8.69
14	.095764995	.17066658	0.56	3.97	.11213326	.18891746	0.59
15	-1.0389004	.17339367	-5.99	4.78	-1.0234861	.19193763	-5.33
16	.0097377673	.17051089	0.06	3.22	-.0026377642	.18874955	-0.01
17	.38152224	.13941371	2.74	4.77	.37820715	.15433049	2.45
18	-.17103404	.13937229	-1.23	4.61	-.17122930	.15428513	-1.11
19	1.1385384	.24087548	4.73	5.30	1.1376905	.26664913	4.27
20	-.62179518	.24087524	-2.58	5.15	-.61980021	.26664877	-2.32
21	-1.2308426	.24289340	-5.07	4.66	-1.2304850	.26888293	-4.58
22	1.2002602	.24289626	4.94	4.78	1.2027073	.26888603	4.47
23	2.0775862	.24087548	8.63	4.63	2.0779142	.26664913	7.79
24	-.23473543	.24088156	-0.97	4.26	-.23712373	.26665574	-0.89
3	-.024742249	.011036817	-2.24	3.35	-.032778077	.012178812	-2.69
2	8.6911535	.95231897	9.13	3.50			

```
RESIDUAL STANDARD DEVIATION =          1.9249792                 2.1309519
   BASED ON DEGREES OF FREEDOM       384-20 = 364              384-19 = 365
```

* THE NUMBER OF CORRECTLY COMPUTED DIGITS IN EACH COEFFICIENT USUALLY DIFFERS BY LESS THAN 1 FROM THE NUMBER GIVEN HERE

Figure D-7. (continued).

APPENDIX E

FIELD TEST DATA

The field tests for the ASTM test tire correlation were conducted at the Texas Transportation Institute. Skid resistance measurements of each set of eight repeat skids were averaged and the variances computed. Tables E-1 list the mean skid numbers by pavement, speed, and time of day for each of four replicates (REPS). The means of the four replicates are also listed. At the bottom of the tables means are listed by speed (pooled for both tires and the four pavements), by site (pooled for the three speeds), and by tire type (pooled for all four pavements and three speeds). The corresponding variances are listed in Tables E-2.

Table E-1. Mean skid number (of 8 repeat skids).

TEST TIRE CORRELATION - MEANS (8 DATA PTS)
UPPER ROW - MORNING, LOWER ROW - AFTERNOON
TEST SERIES 1

		TIRE E249					TIRE E501				
REPS		1	2	3	4	MEAN	1	2	3	4	MEAN
SITE	SPEED										
	20	31.17	30.17	23.21	22.92	26.87	30.60	30.30	26.92	30.42	29.61
		27.55	28.17	24.55	22.42	25.67	30.30	28.80	26.27	25.57	27.74
2	40	16.88	16.61	14.19	18.45	16.53	21.00	19.05	19.07	19.06	19.55
		15.81	15.60	15.32	15.29	15.51	19.74	17.50	16.40	16.60	17.56
	60	11.46	12.21	11.21	11.95	11.71	14.24	14.01	15.55	15.79	14.90
		11.60	11.57	11.14	10.75	11.27	13.89	12.70	13.66	13.91	13.54
	20	28.55	23.19	20.81	20.74	23.32	29.75	22.57	21.84	21.96	24.03
		22.56	21.47	21.16	19.84	21.26	22.32	23.06	23.24	20.90	22.38
11	40	19.80	17.00	16.50	18.94	18.06	22.69	18.34	18.82	18.92	19.69
		16.44	17.00	15.65	16.42	16.38	20.74	18.32	17.32	17.76	18.54
	60	16.61	14.85	13.89	12.94	14.57	18.69	15.56	15.21	16.15	16.40
		13.27	15.42	13.66	13.64	14.00	15.20	15.21	15.91	15.20	15.38
	20	51.55	50.49	49.82	51.04	50.72	52.21	52.46	49.55	50.50	51.18
		43.34	51.27	49.57	52.45	49.16	50.36	50.62	50.11	51.19	50.57
1	40	41.12	40.10	39.05	42.17	40.61	43.59	44.86	43.19	44.42	44.02
		41.89	39.95	38.05	40.63	40.13	43.99	42.79	44.24	42.80	43.45
	60	37.17	37.30	34.92	36.14	36.38	39.95	40.99	39.38	40.26	40.14
		36.67	34.80	35.30	35.42	35.55	40.74	37.56	38.82	38.76	38.97
	20	48.65	38.74	44.14	43.16	43.67	45.40	41.89	40.97	45.01	43.32
		51.04	41.39	40.50	42.32	43.81	42.67	40.00	41.39	40.70	41.19
6	40	36.56	36.17	33.92	35.39	35.51	37.07	38.20	36.42	34.05	36.44
		36.67	32.80	32.92	32.70	33.77	38.07	34.56	34.92	32.42	35.00
	60	35.67	34.17	34.55	33.24	34.41	35.30	32.17	36.51	35.69	34.92
		33.92	32.17	33.55	32.81	33.12	35.92	33.55	36.19	33.64	34.82

BY SPEED	20	40	60
	35.91	28.17	25.01

BY SITE	2	11	1	6
E249	17.93	17.93	42.09	37.38
E501	20.48	19.40	44.72	37.61

BY TIRE	
E249	28.83
E501	30.56

Table E-1. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 2
MEAN

SITE	SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	25.300	22.100	23.700	24.575	24.094	26.537	26.550	31.487	28.600	28.294
		20.625	20.925	20.262	21.012	20.706	27.550	28.100	25.425	24.300	26.344
2	40	13.675	14.912	12.200	14.687	13.869	14.962	21.787	18.062	16.887	17.925
		16.400	15.137	14.362	14.725	15.156	21.050	18.712	18.687	17.012	18.866
	60	11.075	11.600	10.637	11.750	11.266	15.212	15.625	16.050	14.850	15.484
		11.700	11.475	9.025	11.000	10.800	12.575	15.775	15.200	12.550	14.025
	20	20.875	20.550	19.050	21.687	20.541	21.612	24.200	23.825	23.987	23.406
		18.812	19.062	22.350	19.275	19.875	23.050	22.925	22.575	20.625	22.294
11	40	15.962	17.675	14.287	13.737	15.416	16.125	19.737	16.750	18.700	17.828
		15.075	16.650	16.037	15.900	15.916	17.475	19.087	18.687	19.300	18.637
	60	14.525	13.050	12.412	13.250	13.309	15.675	15.450	15.575	16.162	15.716
		12.450	13.425	13.762	13.400	13.259	13.775	15.762	16.050	15.812	15.350
	20	48.575	51.225	51.387	52.200	50.847	47.550	51.500	49.437	53.137	50.406
		49.700	49.437	50.225	52.337	50.425	50.862	51.025	50.237	52.462	51.147
1	40	39.625	40.662	40.500	42.737	40.881	41.337	42.425	42.387	46.100	43.062
		39.875	41.975	41.275	40.875	41.000	42.762	43.550	43.750	45.000	43.766
	60	38.500	38.575	37.375	37.937	38.097	39.175	40.712	39.387	43.712	40.747
		34.225	39.475	36.937	35.675	36.578	38.400	43.187	44.075	41.125	41.697
	20	38.325	37.937	45.100	38.950	40.078	37.350	38.262	43.612	39.962	39.797
		40.212	35.925	35.350	38.600	37.522	44.800	48.137	40.712	36.937	42.647
6	40	33.462	32.325	34.937	32.187	33.228	31.737	32.725	34.450	37.862	34.194
		33.562	38.550	31.862	32.175	34.037	34.012	35.612	33.300	32.925	33.962
	60	33.725	32.087	31.475	34.925	33.053	32.250	34.175	31.225	37.050	33.675
		34.687	36.162	33.050	34.375	34.569	36.200	35.800	36.000	35.300	35.825

BY SPEED	20	40	60
	34.276	27.359	25.216

BY SITE	2	11	1	6
E249	15.982	16.386	42.971	35.415
E501	20.156	18.672	45.137	36.683

BY TIRE	
E249	27.688
E501	30.212

Table E-1. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 3
MEAN

SITE	KEPS SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	25.525	29.175	24.800	26.112	26.403	28.187	31.825	29.300	28.162	29.369
		23.100	28.550	30.175	25.325	26.787	22.462	30.050	32.050	27.337	27.975
2	40	13.062	18.450	14.725	15.700	15.484	14.725	18.937	16.875	16.525	16.766
		13.525	14.112	15.550	15.400	14.647	14.112	17.837	15.812	17.862	16.406
	60	12.300	10.637	10.112	10.000	10.762	12.800	11.962	12.912	11.625	12.325
		11.125	11.825	10.637	11.112	11.175	9.362	11.462	11.612	13.175	11.403
	20	21.662	22.975	20.750	22.237	21.906	24.725	23.325	25.550	22.725	24.081
		22.575	22.000	22.337	19.800	21.678	23.950	23.212	22.937	22.050	23.037
11	40	15.387	14.125	15.067	15.000	14.900	16.725	17.112	17.225	14.887	16.487
		14.600	15.800	15.075	15.475	15.237	16.037	16.987	16.137	17.075	16.559
	60	11.825	11.450	11.475	12.325	11.769	13.412	12.462	13.637	13.600	13.278
		14.375	12.937	11.000	12.787	12.775	11.962	13.287	12.575	13.537	12.841
	20	49.700	49.037	50.237	49.687	49.666	47.625	47.837	50.625	48.537	48.656
		49.300	52.737	49.562	50.100	50.425	51.025	51.137	49.550	49.187	50.225
1	40	39.587	39.837	39.712	40.237	39.844	40.237	42.550	41.762	42.162	41.678
		38.850	41.525	41.300	39.325	40.250	39.975	43.050	42.637	42.150	41.953
	60	33.925	37.425	34.550	34.550	35.112	33.800	38.812	35.312	37.175	36.275
		35.062	37.687	33.925	35.562	35.559	34.550	38.075	38.125	38.475	37.306
	20	35.050	38.712	36.625	33.925	36.128	37.575	37.275	35.437	35.050	36.334
		35.550	34.175	34.537	35.950	35.053	35.050	34.087	34.025	34.800	34.491
6	40	30.675	31.800	28.675	30.925	30.519	34.050	31.800	31.050	32.300	32.300
		30.675	30.675	30.900	28.675	30.231	31.175	33.425	34.050	29.800	32.112
	60	30.550	33.800	31.175	27.800	30.831	28.300	31.425	31.800	28.175	29.925
		27.675	30.550	28.650	27.675	28.637	28.925	31.550	28.850	28.300	29.406

BY SPEED	20	40	60
	33.888	25:961	22.461

BY SITE	2	11	1	6
E249	17.543	16.378	41.809	31.900
E501	19.041	17.714	42.682	32.428

BY TIRE	
E249	26.908
E501	27.966

Table E-1. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 4
MEAN

SITE	SPEED	TIRE E249 1	TIRE E249 2	TIRE E249 3	TIRE E249 4	TIRE E249 MEAN	TIRE E501 1	TIRE E501 2	TIRE E501 3	TIRE E501 4	TIRE E501 MEAN
	20	24.300	23.562	21.800	22.587	23.062	24.300	24.800	23.175	23.950	24.056
		25.925	22.587	22.950	22.275	23.434	25.437	20.850	25.200	24.450	23.984
2	40	12.687	15.700	14.275	13.950	14.153	15.800	17.100	15.525	13.950	15.594
		14.250	15.425	14.600	14.362	14.659	15.787	15.900	16.150	15.300	15.784
	60	10.987	11.262	9.862	10.800	10.728	11.500	10.925	11.225	12.012	11.416
		11.500	11.950	10.637	10.800	11.222	11.712	12.575	11.725	11.262	11.819
	20	23.350	19.412	21.175	21.662	21.400	26.312	20.887	21.800	22.125	22.781
		21.487	20.737	22.112	20.425	21.191	22.600	22.812	26.800	21.525	23.434
11	40	15.437	15.200	15.525	16.050	15.553	17.000	16.400	17.237	16.500	16.784
		15.787	15.787	16.262	15.500	15.834	16.275	16.400	18.937	17.550	17.291
	60	12.562	11.750	11.962	11.400	11.919	12.212	11.862	12.800	13.200	12.519
		11.837	12.550	11.350	10.875	11.653	12.812	12.575	13.537	11.100	12.506
	20	51.012	49.437	48.650	50.987	50.022	48.012	49.437	48.387	48.062	48.475
		51.612	49.575	49.437	47.900	49.631	52.562	51.937	49.437	48.387	50.581
1	40	38.662	38.500	38.562	37.087	38.203	40.075	39.712	37.512	39.800	39.275
		39.637	38.725	37.575	37.087	38.256	39.025	40.500	37.562	38.300	38.847
	60	32.050	30.250	29.425	27.950	29.919	35.925	31.487	27.550	30.875	31.459
		33.525	32.550	30.675	30.300	31.762	32.687	35.337	28.800	31.887	32.178
	20	37.437	43.562	32.675	35.987	37.416	33.437	38.375	33.425	35.512	35.187
		33.112	36.962	33.550	33.550	34.294	33.500	36.825	34.425	37.525	35.569
6	40	28.675	32.462	30.050	30.000	30.297	29.775	31.800	30.987	30.612	30.794
		29.512	32.175	28.925	28.600	29.803	30.425	32.300	29.175	29.862	30.441
	60	30.175	27.062	27.925	26.412	27.894	29.550	28.150	27.300	27.000	28.000
		25.425	27.800	27.925	28.300	27.362	27.612	30.300	27.800	26.462	28.044

BY SPEED	20	40	60
	32.782	25.098	20.650

BY SITE	2	11	1	6
E249	16.210	16.258	39.632	31.178
E501	17.109	17.553	40.136	31.339

BY TIRE	
E249	25.819
E501	26.534

Table E-1. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 5
MEAN

SITE	SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	23.200	22.212	22.462	25.187	23.266	23.200	25.425	24.550	24.300	24.369
		22.525	22.450	26.050	24.550	23.894	22.912	24.050	31.050	28.925	26.734
2	40	15.150	16.275	16.387	16.262	16.019	15.450	17.975	17.475	17.112	17.003
		15.425	16.375	14.012	15.675	15.372	17.362	18.450	15.912	16.887	17.153
	60	11.700	13.125	11.962	11.237	12.006	13.650	15.675	14.850	14.012	14.547
		12.900	12.925	11.362	11.137	12.081	13.162	15.787	14.850	13.162	14.241
	20	23.500	20.987	24.087	22.200	22.694	23.050	24.425	23.050	24.112	23.659
		21.287	20.862	21.362	21.475	21.247	23.725	22.200	24.425	24.687	23.759
11	40	18.325	18.575	17.487	17.600	17.997	19.362	18.450	18.800	18.675	18.822
		16.950	18.337	16.887	16.887	17.266	19.550	18.337	18.200	20.025	19.028
	60	14.100	14.212	15.925	12.575	14.203	16.500	15.537	15.312	14.587	15.484
		13.800	14.112	15.325	12.575	13.953	16.500	15.187	17.237	13.650	15.644
	20	52.787	51.137	49.187	49.300	50.603	52.812	52.450	49.462	49.937	51.166
		52.400	50.350	52.212	49.575	51.134	50.737	52.200	51.937	51.937	51.703
1	40	42.550	40.962	40.262	40.462	41.059	44.812	43.700	41.662	42.437	43.153
		42.050	39.062	42.162	43.075	41.587	42.025	44.625	42.800	42.950	43.100
	60	39.200	36.175	36.637	33.800	36.453	39.475	39.975	37.550	37.500	38.625
		33.425	34.800	34.050	33.800	34.019	37.300	38.975	37.437	38.437	38.037
	20	37.900	37.187	35.225	35.050	36.341	36.737	37.212	41.200	36.437	37.897
		36.062	33.550	39.125	35.700	36.109	38.100	35.075	38.487	35.675	36.834
6	40	31.425	32.050	32.425	30.675	31.644	33.125	33.300	32.925	33.800	33.287
		31.800	31.175	31.550	30.425	31.237	34.925	32.425	33.300	30.800	32.862
	60	31.412	30.300	30.800	28.300	30.203	31.687	31.925	31.425	28.887	30.981
		30.300	29.425	30.425	28.425	29.644	31.175	30.800	31.550	29.800	30.831

BY SPEED	20	40	60
	33.838	27.287	23.810

BY SITE	2	11	1	6
E249	17.106	17.893	42.476	32.530
E501	19.008	19.399	44.297	33.782

BY TIRE	
E249	27.501
E501	29.122

Table E-2. Standard variances (of 8 repeat skids).

TEST TIRE CORRELATION - STANDARD VARIANCES
UPPER ROW - MORNING, LOWER ROW - AFTERNOON
TEST SERIES 1

SITE	REPS SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	3.12	13.98	1.71	0.98	4.95	4.86	15.43	3.27	12.27	8.95
		4.21	1.98	6.95	7.73	5.22	2.57	6.29	4.28	8.35	5.37
2	40	0.41	2.20	1.00	4.09	1.93	5.79	1.46	0.93	2.21	2.60
		0.97	0.85	2.65	2.02	1.62	2.33	1.32	1.05	2.44	1.78
	60	0.47	0.80	0.26	1.53	0.76	0.38	1.11	0.77	1.81	1.02
		1.05	0.27	3.47	0.75	1.38	0.44	0.57	0.50	1.10	0.65
	20	7.64	5.29	3.39	1.46	4.44	6.25	2.12	2.17	1.46	3.00
		1.02	3.26	2.05	4.98	2.83	1.06	2.44	5.96	16.94	6.60
11	40	1.73	0.29	0.36	3.92	1.58	2.07	0.76	0.88	0.52	1.06
		1.20	2.08	0.21	0.27	0.94	1.72	2.14	0.50	0.37	1.18
	60	1.06	1.25	0.44	0.47	0.80	0.44	0.50	0.64	1.46	0.76
		1.17	0.49	0.50	1.34	0.87	0.36	0.64	0.62	0.35	0.49
	20	3.46	10.59	0.93	1.25	4.06	1.09	2.64	1.49	2.48	1.93
		7.31	0.64	0.93	1.41	2.57	3.30	1.23	1.09	3.32	2.23
1	40	2.41	0.87	0.50	0.27	1.01	0.93	1.04	1.77	2.41	1.53
		2.18	0.46	0.79	0.55	1.00	0.32	1.33	2.02	2.82	1.62
	60	1.70	2.29	6.12	1.78	2.97	0.46	1.37	4.27	3.75	2.46
		1.55	4.28	3.14	6.90	3.97	2.26	4.03	4.86	8.88	5.01
	20	9.71	20.47	8.82	18.53	14.38	19.70	18.98	5.38	3.32	11.85
		6.04	7.85	12.14	19.93	11.49	10.96	15.80	7.28	16.71	12.69
6	40	10.05	4.27	1.55	7.11	5.74	13.89	7.05	3.70	6.50	7.78
		4.98	6.86	6.41	4.13	5.60	9.98	10.39	2.70	9.12	8.05
	60	0.41	2.55	2.50	1.87	1.83	0.86	0.84	2.79	6.23	2.68
		2.70	1.41	5.07	15.85	6.26	7.55	3.64	5.86	11.24	7.07

BY SPEED	20 VAR	20 SD	40 VAR	40 SD	60 VAR	60 SD
	6.41	2.53	2.81	1.68	2.44	1.56

BY SITE	2		11		1		6	
E249	2.64	1.63	1.91	1.38	2.60	1.61	7.55	2.75
E501	3.40	1.84	2.18	1.48	2.46	1.57	8.35	2.89

BY TIRE		
E249	3.68	1.92
E501	4.10	2.02

Table E-2. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 2
STANDARD VARIANCES

SITE	REPS SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	4.285	2.857	17.777	18.991	10.978	1.194	0.786	4.459	4.751	2.797
		6.452	4.554	1.337	1.473	3.457	13.071	4.702	0.267	4.285	5.581
2	40	3.962	1.504	5.494	5.124	4.021	2.140	3.016	3.883	8.139	4.294
		5.109	2.694	0.466	0.756	2.256	4.009	3.553	0.439	1.087	2.272
	60	1.214	1.049	0.449	0.680	0.848	1.341	0.696	0.954	0.703	0.924
		0.571	2.716	5.316	0.703	2.327	2.128	0.839	2.914	2.149	2.008
	20	2.174	1.643	2.349	1.016	1.795	3.381	7.506	1.060	3.715	3.916
		4.970	2.166	13.117	0.268	5.130	1.643	2.839	3.259	1.288	2.257
11	40	0.397	1.642	1.019	16.351	4.852	0.280	2.403	0.920	0.989	1.148
		1.059	0.786	0.774	0.906	0.881	0.962	5.210	1.257	2.163	2.406
	60	1.748	0.920	1.041	0.517	1.057	1.516	1.071	1.877	1.174	1.410
		0.786	2.174	3.108	0.703	1.693	1.028	2.766	1.850	4.687	2.585
	20	2.553	1.520	1.658	2.626	2.089	1.150	7.452	2.458	5.341	4.100
		1.605	0.538	0.632	9.168	2.986	0.591	1.464	5.368	3.299	2.686
1	40	5.982	1.436	0.857	8.045	4.080	5.520	3.531	0.889	1.146	2.771
		1.268	3.118	2.243	3.377	2.502	1.425	2.720	2.993	1.548	2.172
	60	8.275	4.375	4.554	4.783	5.497	0.576	3.710	10.403	7.458	5.537
		0.861	9.114	9.097	2.981	5.513	1.166	5.775	6.579	2.917	4.109
	20	20.065	24.376	7.689	20.555	18.171	21.718	6.667	4.524	7.204	10.028
		8.758	4.123	20.096	13.566	11.636	10.513	4.729	14.807	5.095	8.786
6	40	2.939	1.520	9.061	1.405	3.731	6.825	2.189	8.035	14.093	7.785
		7.050	8.272	10.285	3.124	7.183	2.019	1.065	1.999	1.552	1.659
	60	1.314	1.528	4.927	4.695	3.116	3.622	5.409	1.267	1.359	2.914
		3.114	1.767	5.928	2.817	3.406	2.147	3.999	4.401	4.857	3.851

BY SPEED	20	40	60
	6.025	3.376	2.925

BY SITE	2	11	1	6
E249	3.981	2.568	3.778	7.874
E501	2.979	2.287	3.562	5.837

BY TIRE	
E249	4.550
E501	3.667

Table E-2. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 3
STANDARD VARIANCES

SITE	SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	13.956	1.410	6.857	3.652	6.469	23.764	0.393	3.713	1.060	7.232
		11.617	3.928	5.981	12.316	8.461	5.246	3.356	2.214	8.831	4.912
2	40	0.380	0.774	3.905	0.232	1.323	1.299	1.060	1.443	2.203	1.501
		2.059	2.321	1.317	0.571	1.567	5.893	2.206	4.061	4.131	4.073
	60	7.489	1.248	1.947	2.114	3.200	1.049	1.006	3.858	2.685	2.149
		2.434	0.268	0.448	0.801	0.988	0.774	0.751	1.813	1.831	1.292
	20	8.054	12.371	2.834	3.863	6.780	9.482	6.402	7.357	7.694	7.734
		3.488	7.306	2.989	1.143	3.731	6.234	6.353	4.066	0.786	4.360
11	40	1.150	0.928	1.630	0.703	1.103	1.385	3.333	1.260	0.856	1.708
		2.294	1.531	0.516	1.188	1.383	3.666	1.833	0.380	1.934	1.953
	60	0.554	0.966	0.716	2.956	1.298	2.298	1.008	4.771	0.934	2.253
		1.205	1.294	0.934	0.516	0.987	1.308	1.941	1.585	2.323	1.789
	20	0.949	2.637	6.651	3.739	3.494	10.878	3.604	10.052	2.194	6.682
		2.625	8.301	2.141	2.026	3.773	8.485	1.377	6.011	7.507	5.845
1	40	1.468	0.946	4.311	6.593	3.330	0.933	4.237	2.208	6.812	3.548
		6.520	4.876	3.283	4.718	4.849	6.944	3.211	3.541	0.502	3.549
	60	1.838	2.270	2.765	2.214	2.277	4.856	3.489	15.588	0.841	6.193
		8.094	7.133	12.656	10.394	9.569	5.928	4.946	5.554	23.748	10.044
	20	6.213	7.225	15.430	7.267	9.034	8.718	2.996	14.766	7.356	8.459
		5.071	4.552	9.418	13.863	8.226	8.498	1.920	9.124	1.999	5.385
6	40	1.267	3.713	2.695	1.553	2.307	8.213	0.856	3.642	3.142	3.963
		1.838	8.981	1.666	2.695	3.795	0.839	7.124	47.453	2.285	14.425
	60	3.357	0.857	0.553	2.857	1.906	7.713	0.552	2.857	0.553	2.919
		3.267	3.642	2.145	1.267	2.580	9.838	2.499	3.746	1.999	4.520

BY SPEED	20	40	60
	6.286	3.399	3.373

BY SITE	2	11	1	6
E249	3.668	2.547	4.549	4.641
E501	3.527	3.300	5.977	6.612

BY TIRE	
E249	3.851
E501	4.854

Table E-2. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 4
STANDARD VARIANCES

		TIRE E249					TIRE E501				
SITE	REPS SPEED	1	2	3	4	MEAN	1	2	3	4	MEAN
	20	2.285	6.965	0.286	2.539	3.019	4.000	0.571	2.125	0.592	1.822
		14.410	2.639	4.006	2.488	5.886	9.823	2.146	21.734	12.557	11.565
2	40	0.816	1.263	2.411	2.649	1.784	1.734	4.883	0.982	1.826	2.356
		1.666	0.485	2.294	3.551	1.999	3.044	1.449	1.177	1.234	1.726
	60	0.438	1.186	0.880	1.543	1.012	2.020	1.605	3.488	4.724	2.959
		4.340	1.246	2.280	1.131	2.249	2.078	0.839	1.345	2.817	1.770
	20	10.003	0.816	2.125	4.177	4.280	23.844	8.087	1.714	5.060	9.676
		2.696	1.717	7.124	2.488	3.506	5.988	2.801	13.142	2.351	6.071
11	40	1.248	1.789	1.839	4.912	2.447	1.603	2.080	2.831	1.235	1.937
		0.981	0.438	0.751	0.686	0.714	5.522	1.334	6.694	1.415	3.741
	60	1.623	1.191	0.503	0.720	1.009	2.864	0.380	1.334	2.777	1.839
		1.080	1.034	0.920	0.319	0.838	3.664	3.105	3.706	1.646	3.030
	20	3.033	9.065	2.807	5.766	5.168	4.599	3.482	1.830	7.456	4.342
		13.262	3.450	3.658	14.488	8.714	6.545	12.153	3.782	24.210	11.672
1	40	5.578	1.143	3.454	2.286	3.115	2.089	8.658	16.617	7.406	8.693
		9.539	10.094	7.547	5.446	8.156	13.146	14.570	8.912	5.244	10.468
	60	7.641	3.720	2.553	5.281	4.799	3.267	5.287	14.499	14.107	9.290
		6.381	7.927	16.695	2.095	8.275	12.645	6.193	15.999	5.787	10.156
	20	3.225	16.104	6.981	3.532	7.461	2.381	8.554	5.410	5.927	5.568
		7.658	25.264	8.784	10.071	12.945	6.333	18.318	9.123	11.478	11.313
6	40	4.981	3.309	2.214	3.877	3.595	5.561	1.142	4.044	3.556	3.576
		3.267	1.410	9.552	2.330	4.140	5.695	4.285	4.553	1.769	4.076
	60	1.982	2.782	0.696	3.661	2.280	1.070	4.263	0.285	3.220	2.210
		2.267	3.429	2.696	2.600	2.748	2.890	4.857	7.428	2.414	4.397

BY SPEED	20	40	60
	7.063	3.908	3.679

BY SITE	2	11	1	6
E249	2,658	2.133	6.371	5.528
E501	3.700	4.382	9.103	5.190

BY TIRE	
E249	4.172
E501	5.594

Table E-2. (continued).

TEST TIRE CORRELATION DATA
UPPER ROW -MORNING, LOWER ROW - AFTERNOON
TEST SERIES 5
STANDARD VARIANCES

SITE	REPS SPEED	TIRE E249 1	2	3	4	MEAN	TIRE E501 1	2	3	4	MEAN
	20	0.823	2.696	3.532	7.987	3.759	5.146	5.124	1.642	7.714	4.907
		2.616	3.292	4.214	12.786	5.727	2.873	1.642	19.070	17.838	10.356
2	40	1.414	1.002	0.801	1.209	1.107	1.414	2.920	3.462	0.930	2.182
		5.608	0.554	1.113	1.856	2.283	5.654	1.603	1.167	1.216	2.410
	60	1.234	0.405	1.023	1.291	0.988	1.620	0.485	0.703	1.113	0.980
		0.823	1.831	0.377	3.494	1.631	0.757	1.470	3.594	0.774	1.649
	20	0.720	0.624	18.453	4.677	6.119	1.209	4.839	1.643	7.781	3.868
		2.716	2.146	3.274	5.771	3.476	0.765	1.906	2.839	8.301	3.453
11	40	14.096	1.028	0.530	1.592	4.312	3.391	0.231	2.480	2.291	2.099
		1.209	1.589	1.961	2.504	1.816	1.772	0.217	2.820	3.016	1.956
	60	0.000	0.581	9.871	0.554	2.751	0.412	1.046	0.981	1.073	0.878
		1.132	0.258	9.159	0.268	2.704	1.097	0.624	2.597	2.146	1.616
	20	3.353	3.297	13.068	17.488	9.302	20.533	5.506	3.509	5.810	8.840
		3.026	3.584	20.494	9.914	9.254	3.108	2.629	9.224	3.118	4.520
1	40	6.346	0.879	17.028	9.617	8.468	2.873	6.683	7.515	14.169	7.810
		23.214	1.126	1.724	1.972	7.009	2.844	1.816	11.257	18.309	8.556
	60	4.623	3.695	10.320	11.713	7.588	3.237	1.088	1.644	17.410	5.845
		5.695	3.999	15.356	4.857	7.477	4.858	1.945	14.082	2.626	5.878
	20	22.305	11.804	10.679	15.346	15.033	11.758	19.671	31.991	28.475	22.974
		14.094	13.928	21.752	20.735	17.627	22.194	28.434	19.532	7.838	19.499
6	40	10.047	6.499	4.553	6.982	7.020	2.307	1.713	3.838	6.285	3.536
		7.427	6.268	9.356	10.267	8.330	4.696	6.267	5.999	1.428	4.598
	60	9.570	4.000	6.570	4.285	6.106	2.126	11.553	10.267	2.353	6.575
		7.427	3.124	6.267	3.696	5.129	1.124	3.999	6.785	5.714	4.406

BY SPEED	20	40	60
	9.295	4.593	3.888

BY SITE	2	11	1	6
E249	2.583	3.530	8.183	9.874
E501	3.747	2.312	6.908	10.264

BY TIRE	
E249	6.042
E501	5.808

APPENDIX F

LABORATORY TEST DATA

The laboratory tests at CALSPAN Corporation were run on TIRF (Tire Research Facility). Test wheel slip was increased continuously from zero to lockup. The coefficients of friction (Normalized Tractive Force) are plotted by computer, and a typical plot is reproduced in Figure F-1. A total of 46 runs were programmed according to the Run Matrix in Figure F-2. Peak friction numbers and skid numbers are listed in Tables F-1 and F-2, respectively. The test conditions for each run can be found in Figure F-2.

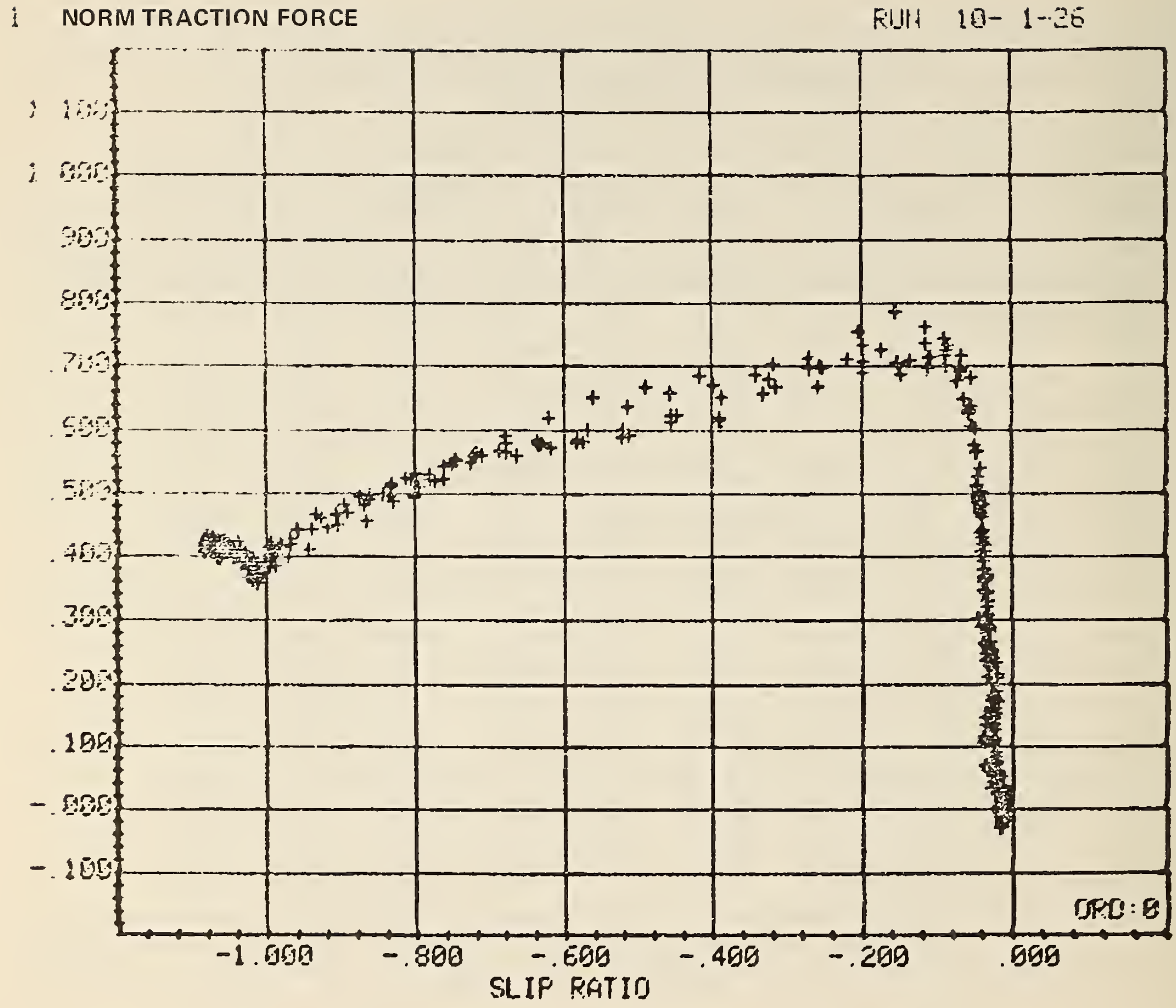

Figure. F-1. Typical computer plot of tire traction versus tire slip.

WATER DEPTH	SKID DEPTH (OF TOTAL)	(OF USABLE)									
VERTICAL LOAD			938 lb	1085 lb						1232 lb	1380 lb
INFLATION PRESS.			24 psi	20 psi	24 psi			28 psi	32 psi	24 psi	24 psi
SPEED			40 mph	40 mph	20 mph	40 mph	60 mph	40 mph	40 mph	40 mph	40 mph
0 mil	95% 70% 35%	100 46 -18				1,(2) 18,(19) 32,(33)					
10 mil	95% 70% 35%					16,(17) 21,(20) 44,(43)					
20 mil	95% 70% 35%		7,(3)	(4)	45,(46)	7,(3) 22,(23) 45,(46)	45,(46)	(5)	(6)	7,(3)	7,(3)
30 mil	95% 70% 35%					8,(9) 25,(24) 41,(42)					
40 mil	95% 70% 35%					11,(10) 26,(27) 40,(39)					
50 mil	95% 70% 35%					12,(13) 29,(28) 37,(38)					
60 mil	95% 70% 35%				35,36,(34)	15,(14) 30,(31) 35,36,(34)	35,36,(34)				

OPEN NUMBERS - STANDARD TEST TIRE ASTM E249   PARENTHESIZED NUMBERS - STANDARD TEST TIRE ASTM E501

Figure F-2. Laboratory test program.

Table F-1. Peak friction numbers.

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
1	96	
	98	
	95	
	99	
2	101	
	102	
	73	
	74	Test 1
	67	
	70	
	73	Test 2
	66	938 lb
	74	
	73	Test 3
	68	
	72	
	73	Test 4
	64	
3	68	
	75	Test 1
	69	
	72	
	70	Test 2
	66	
	71	
	72	Test 3
	67	1085 lb
	69	
	72	Test 4
	67	
	75	
	72	etc.
	67	
	69	
	72	
	68	
	71	
	71	
	64	1232 lb
	70	
	72	
	67	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
	78	1232 lb
3(cont.)	70	(cont.)
	66	
	71	
	69	
	64	
	66	
	69	
	65	1380 lb
	68	
	70	
	64	
	71	
	67	
	63	
	70	
	70	
	66	
	75	
	76	
	74	
	75	
	74	
4	73	
	76	
	76	
	75	
	74	
	73	
	74	
	77	
	79	
	73	
5	72	
	76	
	73	
	72	
	73	
	72	
	74	
	73	
	71	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
5 (cont.)	73	
	71	
	71	
6	75	
	76	
	75	
	74	
	73	
	73	
	70	
	71	
	72	
	70	
	71	
	69	
	70	
	73	
	71	
7	77	938 lb
	83	
	80	
	78	
	80	
	74	
	78	
	73	
	66	
	82	
	73	
	68	
	76	
	72	
	71	
	76	1085 lb
	73	
	67	
	79	
	71	
	68	
	75	
	72	
	68	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
7 (cont.)	74	
	68	
	65	
	77	
	72	
	66	
	72	1232 lb
	70	
	64	
	71	
	69	
	63	
	67	
	70	
	66	
	67	
	68	
	63	
	65	
	67	
	65	
	62	1380 lb
	69	
	61	
	62	
	66	
	62	
	60	
	64	
	61	
	68	
	68	
	61	
	61	
	64	
	60	
8	81	
	83	
	80	
	84	
	80	
	79	

Table F-1. (continued)

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
8 (cont.)	77	
	81	
	75	
	74	
	78	
	75	
	77	
	77	
	71	
9	81	
	85	
	81	
	78	
	82	
	77	
	76	
	78	
	75	
	76	
	78	
	73	
	78	
	75	
	74	
10	76	
	79	
	75	
	75	
	74	
	71	
	73	
	72	
	69	
	71	
	74	
	70	
	71	
	73	
	71	
11	81	
	78	
	77	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
11 (cont)	80	
	77	
	74	
	76	
	77	
	74	
	74	
	74	
	73	
	76	
	74	
	72	
12	76	
	76	
	75	
	75	
	77	
	70	
	70	
	71	
	71	
	70	
	69	
	70	
	71	
	70	
	69	
13	71	
	68	
	70	
	71	
	72	
	68	
	67	
	71	
	70	
	67	
	67	
	67	
	69	
	68	
	67	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
14	69	
	70	
	67	
	68	
	69	
	66	
	64	
	66	
	66	
	66	
	66	
	64	
	64	
	62	
	61	
15	73	
	73	
	73	
	73	
	73	
	70	
	69	
	70	
	70	
	68	
	66	
	66	
	69	
	68	
	68	
16	76	
	77	
	76	
	76	
	73	
	72	
	69	
	73	
	70	
	68	
	69	
	68	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
	71	
	71	
	69	
17	73	
	76	
	74	
	73	
	73	
	70	
	71	
	73	
	71	
	69	
	70	
	71	
	70	
	71	
	68	
18	106	
	104	
	103	
19	104	
	105	
	102	
20	64	
	66	
	62	
	58	
	59	
	59	
	61	
	61	
	61	
	62	
	62	
	62	
	59	
	63	
	58	
21	72	
	75	
	70	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
21 (cont)	65	
	69	
	69	
	67	
	67	
	63	
	64	
	64	
	64	
	65	
	65	
	64	
22	74	
	76	
	75	
	66	
	72	
	68	
	65	
	70	
	70	
	67	
	62	
	64	
	65	
	65	
	64	
23	67	
	73	
	72	
	69	
	68	
	66	
	67	
	67	
	67	
	70	
	70	
	67	
	65	
	64	
	66	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
24	68	
	71	
	67	
	66	
	65	
	64	
	67	
	67	
	66	
	66	
	69	
	68	
	67	
	67	
	67	
25	73	
	73	
	73	
	67	
	70	
	69	
	65	
	64	
	65	
	64	
	67	
	63	
	64	
	62	
	61	
26	66	
	66	
	66	
	64	
	69	
	67	
	64	
	67	
	61	
	61	
	62	
	63	

Table F-1. (continued).

Run No.	F_x/F_z Peak $x10^2$	Remarks
26 (cont)	63	
	61	
	60	
27	65	
	63	
	63	
	62	
	62	
	61	
	65	
	61	
	62	
	63	
	65	
	61	
	61	
	62	
	61	
28	66	
	69	
	68	
	64	
	65	
	64	
	65	
	66	
	65	
	64	
	64	
	62	
	62	
	63	
	62	
29	71	
	70	
	66	
	64	
	67	
	66	
	63	
	64	
	61	

Run No.	F_x/F_z Peak $x10^2$	Remarks
29 (cont)	62	
	64	
	63	
	62	
	64	
	62	
30	63	
	63	
	64	
	60	
	62	
	58	
	61	
	62	
	60	
	59	
	61	
	58	
	60	
	60	
	60	
31	64	
	64	
	63	
	59	
	59	
	57	
	60	
	60	
	61	
	60	
	60	
	57	
	60	
	61	
	60	
32	115	
	114	
	111	
33	116	
	114	
	116	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
34	75	20 mph
	79	
	77	
	73	
	82	
	76	
	76	
	80	
	75	
	70	
	83	
	75	
	78	
	80	
	66	
	44	40 mph
	47	
	43	
	47	
	45	
	40	
	47	
	36	
	45	
	50	
	46	
	40	
	12	60 mph
	8	
	10	
	8	
	11	
	9	
	8	
	10	
	9	
	11	
	10	
	7	
	8	
	8	
	10	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
35	78	20 mph
	85	
	82	
	81	
	86	
	80	
	77	
	79	
	73	
	78	
	88	
	77	
	35	40 mph
	29	
	28	
	33	
	46	
	38	
	48	
	34	
	42	
	50	
	55	
	44	
36	88	20 mph
	92	
	91	
	90	
	82	
	78	
	78	
	82	
	78	
	79	
	85	
	80	
	78	
	83	
	81	
	45	
	52	
	49	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
36 (cont)	30	40 mph
	28	
	25	
	31	
	33	
	34	
	32	
	31	
	35	
	25	
	24	
	22	
	24	
	23	
	21	
	23	
	24	
	23	
	23	
	24	
	22	
37	65	
	63	
	61	
	62	
	64	
	58	
	61	
	64	
	57	
	64	
	62	
	57	
	61	
	63	
	57	
38	50	
	52	
	49	
	49	
	49	
	50	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
38 (cont)	50	
	49	
	49	
	51	
	48	
	48	
	50	
	49	
	47	
39	56	
	55	
	57	
	56	
	54	
	55	
	56	
	54	
	55	
	59	
	59	
	55	
	54	
	53	
	55	
40	70	
	68	
	67	
	66	
	68	
	64	
	65	
	65	
	60	
	67	
	65	
	61	
	60	
	63	
	64	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
41	73	
	72	
	69	
	65	
	66	
	65	
	65	
	67	
	68	
	67	
	65	
	62	
	65	
	64	
	64	
42	60	
	58	
	58	
	58	
	58	
	57	
	60	
	56	
	57	
	60	
	60	
	59	
	57	
	58	
	56	
43	68	
	69	
	67	
	61	
	62	
	62	
	63	
	61	
	60	
	61	
	62	
	63	

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
44	74	
	73	
	71	
	66	
	67	
	65	
	64	
	65	
	63	
	64	
	67	
	63	
	63	
	65	
	63	
45	86	20 mph
	88	
	85	
	76	
	81	
	82	
	80	
	82	
	77	
	77	
	82	
	78	
	76	
	82	
	76	
	53	40 mph
	58	
	56	
	57	
	58	
	60	
	57	
	59	
	58	
	55	
	60	
	58	

Table F-1. (continued).

Run No.	F_x/F_z Peak $\times 10^2$	Remarks
45 (cont)	41	60 mph
	42	
	43	
	44	
	42	
	41	
	45	
	41	
	40	
	41	
	46	
	40	
46	84	20 mph
	89	
	82	
	76	
	84	
	78	
	78	
	87	
	81	
	79	
	87	
	81	
	79	
	84	
	78	
	60	40 mph
	57	
	52	
	58	
	59	
	55	
	59	
	57	
	55	
	62	
	61	
	56	
	42	60 mph
	41	
	43	
46 (cont)	42	60 mph (cont)
	42	
	43	
	42	
	40	
	39	

Table F-2. Skid number, mean skid number, standard deviation and standard deviation of the means.

Run No.	F_x/F_z $\times 10^2$	Remarks
1	82.8	
2	89.0	
	41.7	Test 1
	42.0	Test 2
	41.0	Test 3
	42.7	
	41.9	etc.
Mean →	41.86	938 lb
Std. Dev. →	0.611	
Std. Dev. of Mean →	0.273	
	42.7	
	41.0	
	42.7	
	41.9	1085 lb
	42.6	
	42.18	
	0.7395	
3	0.331	
	40.7	
	40.7	
	42.0	1232 lb
	41.8	
	39.9	
	41.02	
	0.870	
	0.389	
	38.6	
	38.8	
	40.0	1380 lb
	39.0	
	40.4	
	39.36	
	0.792	
	0.354	
	39.8	
	43.0	
	44.0	
4	42.0	
	42.0	
	42.16	
	1.558	
	0.697	

Run No.	F_x/F_z $\times 10^2$	Remarks
	40.0	
	39.4	
	39.2	
	40.3	
5	39.0	
	39.58	
	0.5495	
	0.246	
	39.9	
	39.0	
	38.3	
6	40.0	
	39.0	
	39.24	
	0.709	
	0.317	
	39.4	
	40.3	
	38.9	
	36.9	938 lb
	39.3	
	38.96	
	1.260	
	0.564	
	38.0	
	38.3	
	38.8	1085 lb
7	36.5	
	37.0	
	37.72	
	0.947	
	0.4235	
	36.4	
	36.7	
	35.7	1232 lb
	35.7	
	36.0	
	36.10	
	0.4415	
	0.197	

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks
7 (cont)	35.5	
	34.8	
	35.7	1380 lb
	36.5	
	34.3	
	35.36	
	0.8473	
	0.379	
8	37.0	
	37.0	
	37.0	
	38.9	
	36.4	
	37.26	
	0.9528	
	0.426	
9	39.9	
	41.6	
	41.0	
	41.6	
	39.6	
	40.74	
	0.942	
	0.421	
10	38.0	
	41.5	
	40.1	
	40.0	
	39.8	
	39.88	
	1.248	
	0.558	
11	36.0	
	35.1	
	35.0	
	35.8	
	35.0	
	35.38	
	0.482	
	0.215	

Run No.	F_x/F_z $\times 10^2$	Remarks
12	34.9	
	34.2	
	36.0	
	35.0	
	36.0	-
	35.22	
	0.776	
	0.347	
13	39.6	
	39.7	
	39.6	
	38.0	
	39.0	
	39.18	
	0.716	
	0.320	
14	36.7	
	36.5	
	37.9	
	37.4	
	35.2	
	36.74	
	1.026	
	0.459	
15	34.5	
	35.0	
	34.0	
	34.0	
	34.0	
	34.30	
	0.4472	
	0.200	
16	38.1	
	37.0	
	37.0	
	38.0	
	38.0	
	37.62	
	0.567	
	0.254	

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks
17	42.0 42.0 41.2 42.5 42.5 42.04 0.532 0.238	
18	85.0	
19	89.0	
20	34.4 36.0 36.0 34.0 34.2 34.92 0.996 0.445	
21	32.8 30.2 30.8 33.5 32.1 31.88 1.370 0.613	
22	32.2 31.0 31.0 31.2 31.7 31.42 0.522 0.233	
23	35.8 34.2 35.3 35.6 34.6 35.10 0.678 0.303	
24	36.0 34.4 34.0 34.6 34.0 34.60 0.825 0.369	
25	32.0 32.5 32.0 32.0 32.0 32.10 0.224 0.100	
26	28.7 29.5 30.0 30.0 30.7 29.78 0.740 0.331	
27	33.0 33.0 33.0 33.0 31.9 32.78 0.492 0.220	
28	35.3 33.0 36.0 34.0 34.0 34.46 1.1865 0.531	

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks	Run No.	F_x/F_z $\times 10^2$	Remarks
29	29.2		34 (cont)	7.0	60 mph
	29.2			7.0	
	30.2			7.0	
	30.0			6.0	
	29.2			7.0	
	29.56			6.80	
	0.498			0.4472	
	0.223			0.200	
30	28.6				
	28.5				
	29.0				
	28.2				
	29.1				
	28.68				
	0.370				
	0.166				
31	30.8				
	31.0				
	31.6				
	31.0				
	30.7				
	31.02				
	0.349				
	0.156				
32	86.0				
33	86.0				
34	46.8	20 mph			
	48.2				
	47.0				
	47.4				
	44.8				
	46.84				
	1.260				
	0.564				
	32.2	40 mph			
	31.0				
	31.8				
	23.3				
	29.575				
	4.213				
	2.106				

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks
35	45.0	20 mph
	50.8	
	49.5	
	48.0	
	50.4	
	48.74	
	2.351	
	1.051	
	19.0	40 mph
	18.0	
	23.0	
	21.5	
	20.375	
	2.287	
	1.143	
	11.0	60 mph
36	52.0	20 mph
	50.4	
	51.2	
	50.8	
	49.0	
	50.68	
	1.110	
	0.496	
	29.3	40 mph
	18.2	
	20.0	
	20.0	
	21.875	
	5.022	
	2.511	
	12.0	60 mph
	11.2	
	10.2	
	10.1	
	10.1	
	10.72	
	0.853	
	0.381	

Run No.	F_x/F_z $\times 10^2$	Remarks
35 + 36	45.0	20 mph
	50.8	
	49.5	
	48.0	
	50.4	
	52.0	
	50.4	
	51.2	
	50.8	
	49.0	
	49.71	
	2.012	
	0.636	
	19.0	40 mph
	18.0	
	23.0	
	21.5	
	29.3	
	18.2	
	20.0	
	20.0	
	21.125	
	3.700	
	1.308	
	11.0	60 mph
	12.0	
	11.2	
	10.2	
	10.1	
	10.1	
	10.77	
	0.771	
	0.315	
37	26.0	
	18.2	
	24.8	
	18.0	
	17.0	
	20.80	
	4.245	
	1.898	

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks
38	25.5 28.0 11.8 24.3 11.5	
	20.22 7.937 3.550	
39	32.0 31.0 30.0 26.8 31.0	
	30.16 2.007 0.898	
40	29.5 29.1 28.2 28.0 27.2	
	28.40 0.914 0.409	
41	30.3 29.8 29.8 31.0 30.0	
	30.18 0.502 0.224	
42	24.5 31.0 36.0 28.8 32.0	
	30.46 4.232 1.893	

Run No.	F_x/F_z $\times 10^2$	Remarks
43	35.4 33.0 31.0 35.0 34.5	
	33.78 1.801 0.805	
44	32.0 32.0 32.0 31.0 29.9	
	31.38 0.934 0.418	
45	51.0 52.0 51.0 49.0	20 mph
	50.75 1.258 0.629	
	35.7 34.1 33.4 33.4	40 mph
	34.15 1.085 0.542	
	24.0 22.5 22.8 23.8	60 mph
	23.275 0.737 0.368	

Table F-2. (continued).

Run No.	F_x/F_z $\times 10^2$	Remarks
46	49.4 50.0 51.0 47.1 50.9	20 mph
	49.68 1.587 0.710	
	36.1 35.3 34.9 36.3	40 mph
	35.65 0.661 0.330	
	28.0 28.1 24.5 21.3	60 mph
	25.475 3.248 1.624	

Run No.	F_x/F_z $\times 10^2$	Remarks

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